

THE MATHEMATICS DEPARTMENT PRESENTS

# A MATHEMATICS COLLOQUIUM

THURSDAY, March 12, 2009

BOND HALL 217

4:00 pm

**Title: Normal Form for the Hopf Bifurcation**

**Speaker: Brian Heinze**, Western Washington University

**Abstract:**

A Hopf bifurcation is one in which an equilibrium point of a system of differential equations loses stability as a pair of complex conjugate eigenvalues of the Jacobian matrix cross the imaginary axis. The result is that a spiral sink in the phase portrait becomes a spiral source. One characteristic of the Hopf bifurcation is the presence of a limit cycle. A Hopf bifurcation may be either subcritical (the limit cycle is unstable and is present before the bifurcation) or supercritical (the limit cycle is stable and occurs after the bifurcation). An important theorem in bifurcation theory states that any 2D system of differential equations undergoing a Hopf bifurcation can, under certain conditions, be transformed via coordinate, time and parameter changes into one of two normal forms. Performing this transformation allows one to determine whether the bifurcation is supercritical or subcritical. For practical purposes we just need to compute one coefficient (called the Lyapunov coefficient) of a corresponding complex differential equation to make this determination. In this talk I will outline the proof of this theorem, discuss the two normal forms, and use Rayleigh's equation as an example. The talk will be accessible to anybody with some knowledge of ordinary differential equations.

Refreshments will precede the talk at 3:30pm in Bond Hall 300  
courtesy of Dr. Tilmann Glimm