THE MATHEMATICS DEPARTMENT PRESENTS

A MATHEMATICS COLLOQUIUM

TUESDAY, June 12, 2007
BOND HALL 106
3:00 pm

Title: **Polya Fields**

Speaker: **Selim Akkoc**, Western Washington University

Abstract: A quote from Churchill, *Complex Variables and Applications*: "Definite integrals in elementary calculus can be interpreted as areas and they have other interpretations as well. But except in special cases, no correspondingly helpful interpretation, geometrical or physical is available for integrals in the complex plane." In this talk we demonstrate that this statement is basically incorrect. In fact, the complex contour integral carries with it not one, but two physical interpretations: The real and imaginary parts of the contour integral of a complex function represent the **circulation** and **flux** of the Polya field of the complex function over the contour. (Circulation and flux will be defined during the talk.) The **Polya field** of any complex valued function is the conjugate of the function viewed as a vector field on its domain. For example, the vector field of $f(z) = x + iy$ is $\langle x, y\rangle$ while the Polya field of is $\langle x, -y\rangle$. These fields are pictured below.

We will graph various Polya fields and interpret various contour integrals in terms of the Polya fields of the functions being integrated. A vector field is **incompressible** on a domain if its flow rate (flux) over the boundary of any neighborhood in the domain is zero, and the field is **irrotational** if its circulation over the boundary of any neighborhood in the domain is zero. Our main result is

Theorem: A vector field in the plane is incompressible and irrotational on its domain iff it is the Polya field of an analytic function.

Various real integrals which can be integrated only by complex methods will be examined and reinterpreted from the point of view of Polya fields.

Refreshments will precede the talk at 2:30pm in Bond Hall 300
courtesy of Don Chalice.