

THE MATHEMATICS DEPARTMENT PRESENTS

# A MATHEMATICS COLLOQUIUM

FRIDAY, May 30, 2008

BOND HALL 428

3:00 pm

Title: **Algebraic Number Fields and the Equation**  $x^3 + y^3 = z^3$

Speaker: **Elisabeth Briggs**, Western Washington University

## Abstract:

An algebraic number field is a subfield of the field  $\mathbb{A}$  of algebraic numbers. A quadratic field is of the form  $\mathbb{Q}(\alpha)$ , where  $\alpha$  is a zero of an irreducible quadratic polynomial over  $\mathbb{Q}$ . With some work, one can establish that a quadratic field is of the form  $\mathbb{Q}(\sqrt{d})$ , where  $d$  is a square-free rational integer. We are interested in the case when  $\mathbb{Q}(\sqrt{d})$  is a unique factorization domain. This will be established by means of the theory of Euclidean fields. The goal is to prove that  $x^3 + y^3 = z^3$  has no solutions in positive rational integers using quadratic fields. When Euler first attempted to prove this result he incorrectly assumed that  $\mathbb{Z}[\sqrt{-3}]$  is a unique factorization domain. His second proof used a completely different method, that of infinite descent. In this talk I will prove the result using a unique factorization domain that is actually a field, specifically  $\mathbb{Q}(\sqrt{-3})$ .

Refreshments will precede the talk at 2:30pm in Bond Hall 300  
courtesy of Ms. Sandy Briggs