THE MATHEMATICS DEPARTMENT PRESENTS

A MATHEMATICS COLLOQUIUM

THURSDAY, October 9, 2008
BOND HALL 217
4:00 pm

Title: **Eigenvalue problems with boundary conditions depending polynomially on the eigenparameter**

Speaker: **Branko Curgus**, Western Washington University

Abstract: It is well known that the normalized eigenfunctions of the eigenvalue problem $-f'' = \lambda f,\ f'(-1) = f'(1) = 0$, form an orthonormal basis of the Hilbert space $L^2[-1, 1]$. Also, the eigenfunctions of this problem can be selected to form an orthonormal basis of the Sobolev space $H^1[-1, 1]$.

We will present generalizations of these two statements. For example, let $\mathcal{P}(z)$ be a 2×4 matrix polynomial and let $\mathsf{b}(f)$ be the column vector with the components $f(-1), f(1), f'(-1), f(1)$. We will give sufficient conditions on $\mathcal{P}(z)$ under which the eigenvalue problem $-f'' = \lambda f$, $\mathcal{P}(\lambda)\mathsf{b}(f) = 0$, is equivalent to an eigenvalue problem for a self-adjoint operator $\widetilde{A}$ in a Pontryagin space which is the direct sum of $L^2[-1, 1]$ and a finite-dimensional space. This provides a generalization of the first statement in the abstract. Both, the finite dimensional Pontryagin space and the self-adjoint operator $\widetilde{A}$ are defined explicitly in terms of the coefficients of $\mathcal{P}(z)$. To generalize the second statement we describe the form domain of the operator $\widetilde{A}$.

Instead of the second derivative, our generalizations deal with the adjoint S^* of a closed densely defined symmetric operator S with equal defect numbers $d < \infty$ in a Hilbert space $(\mathfrak{H}, \langle\,\cdot\,,\cdot\,\rangle)$.

This is joint work with Tomas Azizov (Voronezh, Russia) and Aad Dijksma (Groningen, The Netherlands).

Refreshments will precede the talk at 3:30pm in Bond Hall 300.