

WESTERN WASHINGTON UNIVERSITY

# MATHEMATICS COLLOQUIUM

THURSDAY, OCTOBER 29, 2009

Bond Hall 401  
4:00 pm

## Covariograms and the Phase Retrieval Problem

Richard Gardner  
Western Washington University

### ABSTRACT

The Phase Retrieval Problem of Fourier analysis involves determining a function  $f$  on  $\mathbb{R}^n$  from the modulus  $|\hat{f}|$  of its Fourier transform  $\hat{f}$ . The problem arises frequently and naturally in various areas of science, such as X-ray crystallography, electron microscopy, optics, and astronomy, in which only the magnitude of the Fourier transform can be measured and the phase is lost. It is a fundamentally under-determined problem without additional constraints.

One such constraint is that the function  $f$  is the characteristic function  $1_K$  of a convex body  $K$  in  $\mathbb{R}^n$ . In this setting one would like to construct an approximation to  $1_K$  (or, equivalently, to  $K$ ) from a finite number of noisy measurements of  $|\widehat{1_K}|$ .

In this talk, I will explain how this can be done, giving in outline a complete theoretical solution to the problem. The main idea is to utilize a connection to a closely related problem, that of retrieving a convex body from a finite number of noisy measurements of its covariogram. The covariogram  $g_K$  of a convex body gives the volumes of the intersections of  $K$  with its translates, i.e.,

$$g_K(x) = V(K \cap (K + x)), \text{ for } x \in \mathbb{R}^n.$$

The work was done jointly with Gabriele Bianchi (Florence, Italy) and Markus Kiderlen (Aarhus, Denmark). There will be plenty of pictures and, if time allows, some demonstrations of computer programs written by Western undergraduate students funded by the NSF.

Tea at 3:30 in Bond Hall 300  
Refreshments provided by Richard Gardner