THE MATHEMATICS DEPARTMENT PRESENTS

A MATHEMATICS COLLOQUIUM

THURSDAY, January 29, 2009
BOND HALL 217
4:00 pm

Title: **Some Curious Results on Randomly Weighted Self-normalized Sums**

Speaker: **David Mason**, University of Delaware

Abstract: We determine exactly when a certain randomly weighted self-normalized sum converges in distribution, partially verifying a 1965 conjecture of Leo Breiman, and then apply our results to provide a short proof of a 1973 conjecture of Logan, Mallows, Rice and Shepp on the asymptotic distribution of self-normalized sums in the case of symmetry. Specifically, we are interested in sums of the form

$$\frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n Y_i},$$

where $\{Y, Y_i : i \geq 1\}$ denotes a sequence of i.i.d. random variables, Y is non-negative and $\{X, X_i : i \geq 1\}$ is a sequence of i.i.d random variables, independent of $\{Y, Y_i : i \geq 1\}$, and X is in the class $\mathcal{X}$ of random variables X satisfying

$$E|X| < \infty \text{ and } EX = 0.$$

We shall also describe the cluster sets of a special case of these randomly weighted self-normalized sums, namely,

$$\frac{\sum_{i=1}^n s_i Y_i}{\sum_{i=1}^n Y_i},$$

with $\{s, s_i : i \geq 1\}$ being a sequence of independent random signs (i.e. $P\{s = 1\} = P\{s = -1\} = 1/2$ independent of $\{Y, Y_i : i \geq 1\}$.) These are the random sets

$$\mathcal{C} := \bigcap_{n=1}^{\infty} \overline{\left\{ \frac{\sum_{i=1}^m s_i Y_i}{\sum_{i=1}^m Y_i} : m \geq n \right\}}.$$

Along the way, we shall point out an interesting connection between our results in this situation and the classic Saint Petersburg game.
The first part of this talk is based on joint work with Joel Zinn of Texas A&M University.

Refreshments will precede the talk at 3:30pm in Bond Hall 300.