Title: $p$-Adic Numbers and $\mathbb{Q}_p$

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Abstract: In this talk, introducing constructions in analytic number theory, we will examine the rationals under the $p$-adic norm, the completion under this norm, and results that challenge our usual intuition for numbers. The $p$-adic norm ($|\cdot|_p$) on the integers defines two numbers as close if their difference is divisible by a large power of $p$, for the particular prime $p$. This norm extends naturally to the rationals and we will see that all non-trivial norms on $\mathbb{Q}$ are equivalent to a $p$-norm or to the Euclidean norm. We will build $\mathbb{Q}_p$ as the completion with respect to $|\cdot|_p$ and consider properties of $\mathbb{Q}_p$. Working in $\mathbb{Q}_p$ can quickly get messy so we will discuss a process for selecting a unique representative of each equivalence class in $\mathbb{Q}_p$. Consequently any element of $\mathbb{Q}_p$ has a representation in the base $p$. Finally, we will examine a criterion for the existence of roots of polynomials.

Refreshments will precede the talk at 3:30pm in Bond Hall 300 courtesy of Dr. Edoh Amiran.