Egyptians’ Solution of Algebraic Equations

Historical Context:
- When: 3000 – 2000 B.C.
- Where: Egypt
- Who: Scribes et al
- Mathematics focus: Demonstrate use of “rule of false position” to solve algebraic equations.

Suggested Readings:
- Egyptian mathematics: http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Egyptian_mathematics.html
- Egyptian numeration system: http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Egyptian_numerals.html
- 2/n table http://noisefactory.co.uk/maths/history/hist006.html
- NCTM’s Historical Topics for the Mathematics Classroom (1969): “Algebra in Egypt” (p. 237), “Rule of false position” (p. 332), “Equations and the ways they were written” (pp. 260-263), “Early numeration systems” (pp. 22-23), and “Egyptian numeration system” (pp. 38-40).
- Key search words/phrases: Egyptian algebra, rhetorical algebra, rule of false position, Egyptian numeration, Rhind Papyrus, 2/n table

Problem to Explore:
Solve linear equations of form $x + mx = n$ using “rule of false position”

Why This Problem is Important:
- Represents an early stage in the development of algebra.
• Involved use of “rule of false position” which became a powerful numerical technique in modern mathematics.

Problem Solving Experiences:

To Egyptians, linear equations occurred in the form of “aha” or “heap” problems in the *Rhind Mathematical Papyrus* (*RMP*). In the following development, these assumptions are being made:

• Unit fractions will be maintained, but not in the form of Egyptian hieroglyphs
• The symbol x will represent “heap” to simplify things
• You know how to divide and multiply using the Egyptian method

First, the process will be explored using the Egyptian solution of *RMP* Problem 24: 
*Value of heap if heap and a seventh of heap is 19.* The Egyptian process was:

• Assume (falsely?) that \( x = 7 \)
• Then \( x + \frac{1}{7}x = 8 \) not 19 as required

• But by division of 19 by 8, we know \( 19 \div 8 \rightarrow 2 \frac{3}{8} \)

\[
\begin{array}{c|c}
\text{i.e.} & \text{results} \\
\hline
1 & 8 \\
*2 & 16 \\
1/2 & 4 \\
*1/4 & 2 \\
*1/8 & 1 \\
\end{array}
\]

• Thus, the answer is \( x = 7 \left( 2 + \frac{1}{4} + \frac{1}{8} \right) = 16 + \frac{1}{2} + \frac{1}{8} \)

1. Justify the Egyptian process by explaining the reasoning underlying their calculations and logic.
2. Repeat the same problem, but using the “false” guess of \( x = 14 \). Why aren’t \( x = 10, x = 0, \) or even \( x = 70 \) good “false” guesses?

Extension and Reflection Questions:

Extension 1: Using the Egyptian process, solve *RMP* Problem 27: 
*A heap, its fifth added to it so that 21 results.*

Extension 2: Consider *RMP* Problem 28: 
*Two-thirds added and a third [of this sum] taken away so that 10 results.* 
Note: Though the wording of the problem is somewhat confusing, it can be represented symbolically as \( \left( x + \frac{2}{3}x \right) - \frac{1}{3} \left( x + \frac{2}{3}x \right) = 10 \).

Solve this problem using the “rule of false position.”
Extension 3: The Egyptians did not solve the RMP Problem 28 using the “rule of false position.” Rather, the scribe offered this solution: Make one-tenth of this 10: the result is 1, remainder 9. Two thirds of it, namely 6, are added to it; total 15. A third of it is 5. It was 5 that was taken away: remainder 10. Explain what is occurring in this suggested solution.

Extension 4: An important aspect of algebra is the provision of techniques for solving equations of the same type. The “rule of false position” is a useful and general technique, but the Egyptians suggested solution of RMP Problem 28 is both unusual and perhaps suspect.

- Try to use the same technique to solve this slight variation of RMP Problem 28: Two-thirds added and a third [of this sum] taken away so that 20 results. What happens? Modify their technique so it does work.
- Generalize the scribe’s technique to solve this variation of Problem 28: Two-thirds added and a third [of this sum] taken away so that n results.
- Finally, use modern algebra to solve the equation \( x + \frac{2}{3}x - \frac{1}{3}(x + \frac{2}{3}x) = n \) for x. Is there a correspondence between your generalized technique and this new expression for x?

Open-ended Exploration: Using the Egyptian process of “rule of false position,” try to solve RMP Problem 32: A heap, it’s third and it’s fourth added to it so that 2 results. It turns out that the Egyptians did not solve this problem using that process, but did something else that perhaps is more natural. Do research to determine the Egyptians’ solution approach for RMP Problem 32.