Eratosthenes Measure of the Earth’s Circumference

Historical Context:
- When: ca. 240 B.C.
- Where: Alexandria, Egypt
- Who: Eratosthenes
- Mathematics focus: Determination of the circumference of the earth.

Suggested Readings:
- Eratosthenes and his contributions to mathematics, science, and culture: http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Eratosthenes.html
- Ancient geodesy: http://www.absoluteastronomy.com/topics/History_of_geodesy
- NCTM’s Historical Topics for the Mathematics Classroom (1969): “Eratosthenes” (pp. 72-74).
- Key search words/phrases: Eratosthenes, circumference of earth, Alexandria library, Egyptian astronomy, history of geodesy

Problem to Explore:
Determine the circumference of the earth.

Why This Problem is Important:
- Example of the use of proportional reasoning and a simple scientific experiment to make big steps in astronomical calculations.
- Helped establish the idea that the earth’s surface is curved like a circle.

Problem Solving Experiences:
In addition to being chief librarian for the Alexandria library, Eratosthenes was a scholar in mathematics, literature, geography, and history. Though the story of Eratosthenes’ calculation of the circumference of the earth has many versions, most of the stories integrate these assumptions:
- By his observations, Eratosthenes knew that at noon on the longest day of the year (or summer solstice) the sun would be directly overhead the Egyptian city of Syene (now Aswan), which was on the Tropic of Cancer. But, the sun would not be directly overhead Alexandria, which lay to the north of Syene.
• Eratosthenes assumed that the distance between the sun and the earth was so great that one could assume that the sun’s rays were parallel as they “hit” the earth.
• Eratosthenes assumed that the earth’s shape was a sphere (in agreement with Aristotle’s claim).
• Eratosthenes knew that Syene was on the same line of longitude (or meridian) as Alexandria.
• Eratosthenes knew that the distance between Alexandria and Syene was 5040 stadia, where a stadion is a measurement unit for length.

For his experiment, Eratosthenes used two gnomons (straight sticks perpendicular to the earth’s surface, one in Syene and the other in Alexandria. At noon on the summer solstice, the sun cast no shadow at the foot C of the gnomon in Syene, verified by the sun also hitting the very bottom of a deep well in Syene. However, in Alexandria at that same time, the sun cast a shadow AB at the foot of the gnomon A, from which Eratosthenes calculated that the inclination of the sun’s rays to the vertical gnomon to be 1/50th of a circle (perhaps using a plumb hanging from the tip of the gnomon to the tip of the shadow thereby forming angle ADB). The picture is as follows:

1. Use Eratosthenes’ information to calculate the circumference of the earth in stadia.
2. Find the equivalent length of the ancient stadion in feet, then calculate the earth’s circumference in miles. How does this value compare with the “modern” value?

The stories about Eratosthenes vary at this point. Some claim that the inclination angle was 1/48th of a circle, not 1/50th. Some claim that Eratosthenes added 2000 more stadia to his calculated value of stadia, to account for experimental error in his measurements. And, some claim common Eratosthenes used the Greek stadion equal to 0.185 kilometers (i.e. the length of foot races at ancient Olympic games), rather than the expected Egyptian stadion equal to 0.157 kilometers.

3. For all three of these alternative versions, calculate the respective circumferences of the earth in miles.
Extension and Reflection Questions:

Extension 1: List sources of possible error in Eratosthenes experiment, including an analysis of his assumptions.

Extension 2: In the 20th century, Enrico Fermi, a Nobel-prize-winning physicist, was well known for his uncanny ability to make good estimates in calculations involving little or no given data. For example, he is credited with using this intuitive way to calculate the circumference of the earth.
- How many time zones do you pass through when you fly from New York to Los Angeles?
- How many miles is it, about, over that same distance?
- How many miles per time zone, on average?
- How many time zones must there be around the world?
- How many miles around the world?
Answer the questions to determine Fermi’s estimate for the earth’s circumference.

Extension 3: Imagine that the earth is a perfect sphere with a metal wire wrapped around its equator, forming a circle snugly fitting against the earth’s surface. Now, imagine that this wire is cut at some convenient spot, and an additional 30 foot-length of wire is spliced into it. This splice will now cause some slack in the wire to occur, however slight it may be. To equalize this slack, posts of an appropriate height are placed all around the earth’s circumference to hold the wire taut, thereby creating a larger circle than the one prior to the cut. Would you be able to:
- Walk under the wire normally,
- Walk under it but by bending your body down a lot,
- Just barely squeeze under it (if at all), or
- None of the above?
Finally, how would your answer change if you were on a planet with a different radius?

Open-ended Exploration: Challenge students to design and implement an experiment to calculate the earth’s circumference using techniques similar to those of Eratosthenes. Also, investigate Eratosthenes’ adaptation of the Greek astronomer Aristarchus of Samos’ work, so that he could first calculate the diameters of both the moon and the sun and then calculate the distances from the earth to either the moon or the sun.