Eratosthenes Measure of the Earth’s Circumference

Solution Commentary:

Solution of Main Problems:

1. Because of Eratosthenes’s assumption that the sun’s rays were parallel, we know the intercepted angle at the center of the earth is also $1/50^{th}$ of a circle or $7^\circ 12'$. Thus, the distance of 5040 stadia between Alexandria and Syene must equal $1/50^{th}$ the circumference of the earth. The circumference can then be calculated by this proportion:

$$\frac{5040 \text{ stadia}}{x \text{ stadia}} = \frac{1/50^{th} \text{ circumference}}{\text{Earth's circumference}}.$$

Or, the earth’s circumference $x = 252,000$ stadia.

2. If Eratosthenes used the Egyptian stade equal to 0.157 kilometers, $252,000$ statia $\approx 39,564$ kilometers $\approx 24,585$ miles. This value is quite close to the modern value of 24,859.8 miles, with about 1% error.

3. Repeating the original process but using $1/48^{th}$ circumference, $x = 241,920$ stadia $\approx 37,981$ kilometers $\approx 23,601$ miles (or about 5% error). Repeating the original calculation but adding 2000 stadia, $x = 254,000$ stadia $\approx 39,878$ kilometers $\approx 24,778$ miles (or about 0.3% error). Repeating the original process but using the Olympic stade, $x = 252,000$ stadia $\approx 46,620$ kilometers $\approx 28,968$ miles (or more than 16% error).

Extension 1: Some possible sources of error in assumptions and methods:
- Choice of *stadion* used, though this would be accounted for by using a common unit to determine the distance from Syene to Alexandria;
- Syene actually lies about 22 miles north of, not on, the Tropic of Cancer;
- Alexandria is not directly north of Syene;
- The precision of the measurement of the inclination angle (Note: The actual difference in latitude between Alexandria and Syene $7.1^\circ$, not the $7^\circ 2'$ determined by Eratosthenes);
- The method used to determine the distance between Alexandria and Syene, as it is not a straight line walk along the Nile (Note: Lore has it that Eratosthenes trained a soldier to take equal paces, then used him to walk the distance, counting his steps.)


Extension 2: The number of time zones (NY – LA) is 3. The distance (NY – LA) is 3000 miles, implying each time zone has about 1000 miles. Since there are 24 time zones (one for each hour in the day), the earth’s circumference is about 24,000 miles. (Problem source is http://rainbow.ldeo.columbia.edu/courses/v1001/fermi.html)
Extension 3: Though this problem has nothing to do with Eratosthenes and his experiment, it is a problem that all students should face because of the surprise awaiting them. Let $r =$ radius of earth in feet, making its circumference $C = 2\pi r$, which also is the length of the original wire circle. Once the 40 foot length is spliced in, the new length of the wire circle is $2\pi r + 40$. But, this implies that the radius of the new circle is $[2\pi r + 40]/[2\pi] = r + [40]/[2\pi] \approx r + 6$ feet 4 inches. Thus, most people would be able to walk under the wire normally. And, since the general term $r$ was used as the radius, this result is independent of the radius of the planet involved (even if it was the size of a baseball).

Open-Ended Exploration: Experiments will vary. Encourage students to participate in international efforts to combine data in this calculation effort. One example is the Noon Day Project (http://www.ciese.org/noonday/), which occurs twice each year.

Teacher Commentary:

Though the mathematics in these explorations is not difficult, emphasize that Eratosthenes was accomplishing this feat in 240 B.C. Also, raise the issues of precision, approximation, experimental error, significant digits, and choice of measurement units.

A video offers an interesting video of Carl Sagan demonstrating Eratosthenes experiment. His use of paper models clarifies issues such as the role of curvature and the use of shadows. The video is available at many sources such as http://www.youtube.com/watch?v=0JHEqBLG650, http://video.aol.com/video-detail/carl-sagan-eratosthenes-and-the-circumference-of-the-earth/2305843010845213437/?icid=VIDURVHOV01, or http://berto-meister.blogspot.com/2008/05/eratosthenes-and-circumference-of-earth.html.

As a good source of writing projects, students can explore any of the following ideas relative to Eratosthenes and his determination of the earth’s circumference:

- Investigate and document the history of efforts to determine the circumference of the earth. For example, in his On the Heavens II, Aristotle noted that those mathematicoi (i.e. Pythagoreans or followers of Chaldean mathematical sciences) who try “to calculate the circumference by proportional calculations” end up with a circumference equal to 400,000 stadia. While Plato also “guessed” the circumference to be 400,000 stadia, Archimedes estimated the value to be 300,000 stadia. Nicastro (2008) is a good resource.

- Investigate the relationship between the historical development of astronomy and that of mathematics. Was one more of a driving force that impacted the other? Consider all ancient civilizations, including China, Japan, and India. Two contrasting resources are Joseph (2000) and Kline (1959, Chapter 7), while Dreyer (1953) takes a more even approach.
• Investigate and document the history of the development of the circumference and area formulas for circles. For example, Nicastro (2008, pp. 166-167) claims that Aryabhata, an Indian mathematician (476 – 550), established the special between the circumference of a circle and its diameter. Do you agree?
• Eratosthenes is well known for his “sieve” that found prime numbers. Determine the strengths and weaknesses of the process used in his sieve. Also, explore variations on Eratosthenes that have been suggested by others, using resources such as Harman (2007), Cox (2008), Quesada (1997), or Fisher (1929).

Additional References:


