Exponential and Logarithm Rules Reminder Sheet

Here are some facts you’ll be glad you remembered.

**Exponent Rules:** (a & b are positive real numbers, x & y are real numbers)
Write examples of each rule. Can you prove it in general?

\[a^x a^y = a^{x+y}\]

\[\frac{a^x}{a^y} = a^{x-y}\]

\[(a^x)^y = a^{xy}\]

\[(ab)^x = a^x b^x\]

\[\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}\]

If \(a \neq 1\), \(a^x = a^y \iff x = y\)

If \(x \neq 0\), \(a^{-x} = b^{-x} \iff a = b\)

If \(a \neq 0\), \(a^0 = 1\)

**Logarithm Rules:** (a, x, y are positive real numbers, \(a \neq 1\))
Write examples of each rule. Can you prove it in general?

\[\log_a(xy) = \log_a(x) + \log_a(y)\]

\[\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)\]

\[\log_a(x^y) = y \log_a(x)\]

You may not remember this, but it will come in handy when you are asked to graph \(y = \log_a(x)\) and \(a\) is not 10 or e.

\[\log_b x = \frac{\log_c x}{\log_c b}\]

Where \(c\) is any base you choose, (I’d choose 10 or e, since that’s what your calculator does by default)

proof: \(\log_b x = A \Rightarrow b^A = x \Rightarrow \log_c (b^A) = \log_c (x) \Rightarrow A \log_c (b) = \log_c (x) \Rightarrow A = \frac{\log_c (x)}{\log_c (b)}\) q.e.d.