

Exponential and Logarithm Rules Reminder Sheet

Here are some facts you'll be glad you remembered.

Exponent Rules: (a & b are positive real numbers, x & y are real numbers)

Write examples of each rule. Can you prove it in general?

$$a^x a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

$$(ab)^x = a^x b^x$$

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

$$\text{If } a \neq 1, a^x = a^y \Leftrightarrow x = y$$

$$\text{If } x \neq 0, a^x = b^x \Leftrightarrow a = b$$

$$\text{If } a \neq 0, a^0 = 1$$

Logarithm Rules: (a, x, y are positive real numbers, $a \neq 1$)

Write examples of each rule. Can you prove it in general?

$$\log_a(xy) = \log_a(x) + \log_a(y)$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$$\log_a(x^y) = y \log_a(x)$$

You may not remember this, but it will come in handy when you are asked to graph $y = \log_a(x)$ and a is not 10 or e.

$$\log_b x = \frac{\log_c x}{\log_c b}$$

Where c is any base you choose, (I'd choose 10 or e, since that's what your calculator does by default)

$$\text{proof: } \log_b x = A \Rightarrow b^A = x \Rightarrow \log_c(b^A) = \log_c(x) \Rightarrow A \log_c(b) = \log_c(x) \Rightarrow A = \frac{\log_c(x)}{\log_c(b)} \text{ q.e.d.}$$