MATH 125

Assignment 1
January 28, 2009

Name _____

Problem 1. Consider the function

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{for} \quad x \neq 0 \\ \\ 1 & \text{for} \quad x = 0. \end{cases}$$

As it was discussed in Math 124, this function is continuous. Let n be a positive integer and consider the definite integrals

$$I_n = \int_0^{n\pi} f(x)\, dx, \qquad n = 1, 2, 3, \ldots .$$

(a) Use properties of the definite integral discussed in Section 5.4 to arrange the numbers

$$I_1,\ I_2,\ I_3,\ I_4,\ I_5,$$

in increasing order.

(b) Give a detailed justification of your inequalities. State clearly which properties of definite integrals from Chapter 5 you used and how they apply to the definite integrals I_1, I_2, I_3, I_4, I_5.

(c) Do you recognize a pattern in the ordering of the numbers

$$I_1,\ I_2,\ I_3,\ I_4,\ I_5,\ I_6,\ I_7,\ I_8,\ I_9,\ I_{10},\ I_{11},\ \ldots\ ?$$

State this pattern clearly.

Problem 2. Consider the function $g(x) = (\sin x)^2$.

(a) The function g has symmetries which can help you calculate the definite integrals below. Discover these symmetries and explain them.

(b) Use this symmetry, not calculus to evaluate the following two integrals.

$$\int_0^{\pi} g(x)\, dx, \qquad\qquad \int_0^{\pi/2} g(x)\, dx.$$

Provide a clear explanation of your reasoning.

Problem 3. Let a be a positive number; for example $a = 1/2$, or $a = 1$, or $a = 2$. The goal of this problem is to study the following definite integrals

$$I(a) = \int_0^1 \sin(2\pi x^a)\, dx, \quad a > 0.$$

(a) Identify all values of a for which $I(a)$ is positive. Identify all values of a for which $I(a)$ is negative. Explain your reasoning.

(b) Using your calculator or the Excel spreadsheet that I posted on my website make the table of values of $I(a)$ for the following values of a

0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 2 3 4 5 6 7 8 9 10

Based on this table sketch the graph of the function $I(a)$.

(c) Can you guess the values of $\lim_{a\to 0} I(a)$ and $\lim_{a\to\infty} I(a)$? Explain your reasoning.

(d) Use the above table to find approximations for the maximum possible value of $I(a)$ and the minimum possible value of $I(a)$. Explain why you think that the values that you found are good approximations. Calculating few more values of $I(a)$ will help strengthen your argument.