Today, Thursday, January 29, 2009, we did Integration by Substitution. Since this is just the integral version of the Chain Rule, and since our nickname for the Chain Rule is the "Onion Rule", the Integration by Substitution could be called the "Reversed Onion Rule".

I illustrated the "Reversed Onion Rule" by calculating

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = \left| \begin{array}{l} w = 1 - x^2 \\ \dfrac{dw}{dx} = -2x \\ dw = -2x dx \\ x dx = -\frac{1}{2}\, dw \end{array} \right|$$

$$\begin{aligned} &= -\frac{1}{2}\int \frac{1}{\sqrt{w}}\,dw \\ &= -\frac{1}{2}\, 2\sqrt{w} + C \\ &= -\sqrt{1-x^2} + C. \end{aligned}$$

Then we celebrated by using the "Onion Rule":

$$\frac{d}{dx}\Big(-\sqrt{1-x^2} + C\Big) = -\frac{1}{2}\big(1-x^2\big)^{-1/2}(-2x) = \frac{x}{\sqrt{1-x^2}}.$$

But, at this point, be aware of a possible despair!

Now I will remind you that

$$\frac{d}{dx}\big(\arctan x\big) = \frac{1}{1+x^2}.$$

So, is it now easy to calculate

$$\int \frac{x}{1+x^4}\,dx\ ?$$

Tomorrow, we will do definite integrals. For example the following one is interesting:

$$\int_0^1 x\sqrt{1-x^2}\,dx.$$

What is the geometric meaning of this definite integral? You can use this geometric interpretation and Math 124 to get a quite good upper estimate for this definite integral.