Exercise 9

The first sequence turns out to be difficult to find a formula for. I quessed this formula based on some other sequences that I studied.

```
Clear[a, n];
a[n_] :=
  If[n - Floor[1/2 (1 + Sqrt[4 n - 3])] (Floor[1/2 (1 + Sqrt[4 n - 3])] - 1) <= Floor[1/2 (1 + Sqrt[4 n - 3])], 1, 0];

Table[a[n], {n, 1, 50}]
```

```
{1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1,
 1, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 0}
```

```
Clear[b, n];

b[1] = 1; b[2] = 2;

b[n_] := b[n] = If[
   Or[OddQ[b[n - 1]] == True, b[n - 1] == b[n - 2]], b[n - 1] + 1, b[n - 1]]

Table[b[n], {n, 1, 23}]
```

```
{1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, 9, 10, 10, 11, 12, 12, 13, 14, 14, 15, 16}
```

```
Clear[c, n];

c[n_] := If[OddQ[n] == True, 0, 2^(n/2)]

Table[c[n], {n, 0, 23}]
```

```
{1, 0, 2, 0, 4, 0, 8, 0, 16, 0, 32, 0, 64, 0, 128, 0, 256, 0, 512, 0, 1024, 0, 2048, 0}
```

```
Clear[d, n];

d[n_] := 3 * 2^n

Table[d[n], {n, 0, 23}]
```

```
{3, 6, 12, 24, 48, 96, 192, 384, 768, 1536, 3072, 6144, 12288, 24576, 49152,
 98304, 196608, 393216, 786432, 1572864, 3145728, 6291456, 12582912, 25165824}
```

```
Clear[e, n];

e[n_] := 15 - n * 7

Table[e[n], {n, 0, 23}]
```

```
Clear[f, n];
f[1] = 3;

f[n_] := f[n] = n + f[n - 1]

Table[f[n], {n, 1, 23}]

Clear[g, n];

g[n_] := 2 * n^3

Table[g[n], {n, 1, 23}]

Clear[h, n];

h[n_] := n! + 1

Table[h[n], {n, 1, 23}]
```

```
{2, 3, 7, 25, 121, 721, 5041, 40321, 362881, 3628801, 39916801,
 479001601, 6227020801, 87178291201, 1307674368001, 20922789888001,
 355687428096001, 6402373705728001, 121645100408832001, 2432902008176640001,
 51090942171709440001, 1124000727777607680001, 25852016738884976640001}
```

Exercise 10

```
Clear[a, n];

a[1] = 3;

a[n_] := a[n] = (2 n - 1) + a[n - 1]

Table[a[n], {n, 1, 23}]

Clear[b, n];

b[n_] := 7 + n * 4

Table[b[n], {n, 0, 23}]

Clear[c, n];

c[n_] := BaseForm[n, 2]

Table[c[n], {n, 1, 23}]
```

$\{1_2, 10_2, 11_2, 100_2, 101_2, 110_2, 111_2, 1000_2, 1001_2, 1010_2, 1011_2, 1100_2, 1101_2,$
$1110_2, 1111_2, 10000_2, 10001_2, 10010_2, 10011_2, 10100_2, 10101_2, 10110_2, 10111_2\}$

```
Simplify[(Sqrt[n - 1] - 1)^2]
```

$-2\sqrt{-1+n}+n$

```
Clear[d, n];

d[1] = 1;  d[2] = 2;

d[n_] := d[n] = If[IntegerQ[Sqrt[n - 1]] == True, d[n - 1] + d[-2 Sqrt[-1 + n] + n], d[n - 1]]

Table[d[n], {n, 1, 50}]
```

```
{1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, 8, 8, 8, 8, 8, 8, 8, 8, 8, 13, 13, 13, 13,
 13, 13, 13, 13, 13, 13, 13, 13, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 21, 34}
```

```
Clear[e, n];

e[1] = 0;

e[n_] := e[n] = 2 + 3 * e[n - 1]

Table[e[n], {n, 1, 50}]

Clear[f, n];

f[1] = 1;

f[n_] := f[n] = (2 n - 1) * f[n - 1]

Table[f[n], {n, 1, 23}]
```

This would be hard without the known formula for the repeat sequence

```
Clear[g, n];

g[n_] := Mod[Floor[1/2 + Sqrt[2 n]], 2]

Table[g[n], {n, 1, 50}]
```

```
{1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 1, 1, 1,
 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0}
```

```
Clear[h, n];

h[1] = 2;

h[n_] := h[n] = h[n - 1]^2

Table[h[n], {n, 1, 7}]
```

```
{2, 4, 16, 256, 65536, 4294967296, 18446744073709551616}
```

Exercise 32

■ (a)

```
Clear[a, n];

a[n_] := 3 * Floor[n / 2] + n - 2 Floor[n / 2] + 1

Table[a[n], {n, -12, 12}]
```

```
{-17, -16, -14, -13, -11, -10, -8, -7, -5,
 -4, -2, -1, 1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19}
```

```
Clear[a, n];

a[n_] := Floor[3 n / 2] + 1

Clear[ai, n];

ai[n_] := n - Floor[ n / 3] - 1

ai[a[13]]
```

```
13
```

```
Clear[ai, n];

ai[n_] := Ceiling[2 n / 3] - 1

Table[{n, ai[n]}, {n, -12, 12}]
```

```
{{-12, -9}, {-11, -8}, {-10, -7}, {-9, -7}, {-8, -6}, {-7, -5}, {-6, -5},
 {-5, -4}, {-4, -3}, {-3, -3}, {-2, -2}, {-1, -1}, {0, -1}, {1, 0}, {2, 1},
 {3, 1}, {4, 2}, {5, 3}, {6, 3}, {7, 4}, {8, 5}, {9, 5}, {10, 6}, {11, 7}, {12, 7}}
```

```
p = 2; q = 3; Table[{n, Ceiling[p/q Floor[q/p n]]}, {n, -12, 12}]
```

```
{{-12, -12}, {-11, -11}, {-10, -10}, {-9, -9}, {-8, -8}, {-7, -7}, {-6, -6},
 {-5, -5}, {-4, -4}, {-3, -3}, {-2, -2}, {-1, -1}, {0, 0}, {1, 1}, {2, 2}, {3, 3},
 {4, 4}, {5, 5}, {6, 6}, {7, 7}, {8, 8}, {9, 9}, {10, 10}, {11, 11}, {12, 12}}
```

```
p = 5; q = 7; b = 2; Table[{n, Ceiling[p/q (Floor[q/p n] + b)] - b}, {n, -12, 12}]
```

```
{{-12, -12}, {-11, -12}, {-10, -10}, {-9, -9}, {-8, -9}, {-7, -7}, {-6, -7},
 {-5, -5}, {-4, -4}, {-3, -4}, {-2, -2}, {-1, -2}, {0, 0}, {1, 1}, {2, 1}, {3, 3},
 {4, 3}, {5, 5}, {6, 6}, {7, 6}, {8, 8}, {9, 8}, {10, 10}, {11, 11}, {12, 11}}
```

```
p = 5; q = 7; b = 1; Table[(Floor[q/p n] + b), {n, 0, 23}]
```

```
{-1, -1, -1, -2, -2, -3, -3, -3, -4, -4,
 -5, -5, -5, -6, -6, -7, -7, -7, -8, -8, -9, -9, -9, -10}
```

■ (b)

```
Clear[b, n];
```

```
b[n_] := (-1)^n 5 (7 * Floor[n / 12] + Mod[Floor[n / 2], 6] + 1)
```

```
Table[b[n], {n, 0, 43}]
```

```
Table[n + Floor[n/6] + 1, {n, -10, 10}]
```

```
{-11, -10, -9, -8, -6, -5, -4, -3, -2, -1, 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12}
```