21

▪ a)

```
5!
```

```
120
```

▪ b)

```
4!
```

```
24
```

▪ c)

```
5!
```

```
120
```

▪ d)

```
4!
```

```
24
```

▪ e)

```
3!
```

```
6
```

▪ f)

```
0
```

```
0
```

23

```
Binomial[9, 5] 5 ! 8 !
```

```
609638400
```

```
(* no consecutive 0 s in bit strings of length  13 with exactly 5 0 s *)
```

```
Binomial[9, 5]
```

```
126
```

```
2^13
```

```
8192
```

■ Below is an exercise in *Mathematica* list manipulation to confirm the above calculation

All bit strings of length 13:

```
tt1 = PadLeft[#, 13] & /@ Table[IntegerDigits[k, 2], {k, 0, 2^13 - 1}];
```

```
Length[tt1]
```

```
8192
```

```
tt1⟦8192⟧
```

```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

```
tt1⟦234⟧
```

```
{0, 0, 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 1}
```

Separate 0s and 1s by sorting:

```
Split[Sort[tt1⟦123⟧]]
```

```
{{0, 0, 0, 0, 0, 0, 0, 0}, {1, 1, 1, 1, 1}}
```

```
Split[Sort[tt1⟦8192⟧]]
```

```
{{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}}
```

```
Split[Sort[tt1⟦1⟧]]
```

```
{{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

The last two bit strings are problematic so exclude them

```
And[(Length[Split[Sort[#]]] == 2), Length[Split[Sort[#]][[1]]] == 5] &[tt1[[123]]]
```

```
False
```

Select the bit strings with 5 0s:

```
tt2 = Select[tt1, And[(Length[Split[Sort[#]]] == 2), Length[Split[Sort[#]][[1]]] == 5] &];
```

```
Length[tt2]
```

```
1287
```

```
Binomial[13, 5]
```

```
1287
```

```
tt3 = Map[Split, tt2];
```

```
Length[tt3]
```

```
1287
```

```
tt3[[123]]
```

```
{{0, 0}, {1, 1, 1}, {0}, {1, 1}, {0}, {1, 1}, {0}, {1}}
```

Now somewhat complicated function to select only those strings that have no consecutive 0s:

```
MyF22[lll_] := Apply[And, Map[Or[And[(#[[1]] == 0), Length[#] == 1], (#[[1]] == 1)] &, lll]]
```

```
MyF22[tt3[[123]]]
```

```
False
```

```
MyF22[tt3[[445]]]
```

```
False
```

```
tt4 = Select[tt3, MyF22[#] &];
```

```
Length[tt4]
```

```
126
```

```
Binomial[9, 5]
```

```
126
```

25

```
(* a *) 100 99 98 97
```

```
94109400
```

```
(* b *) 99 98 97
```

941094

```
(* c *)4 99 98 97
```

3764376

```
(* d *) 99 98 97 96
```

90345024

```
(* e *) 2! Binomial[4, 2] 98 97
```

114072

```
(* f *) 3! Binomial[4, 3] 97
```

2328

```
(* g *) 4!
```

24

```
(* h *) 96 95 94 93
```

79727040

```
(* i *) 4 99 98 97
```

3764376

```
(* j *) 2 Binomial[4, 2] 96 95
```

109440

29

■ (a)

```
(* there are 98 consecutive triples which can be accompanied to make a 4-
  permutation by 97 remaining numbers,
 the accompaniment being possible in 4 different ways *)
```

```
98*97*4
```

38024

```
(* take into account that we count 1,2,
 3,4 twice, and there are 97 such permutations  *)
```

```
98 * 97 * 4 - 97
```

37927

■ (b)

```
(* similar as (a) *)
```

```
98 * 72 * 2 - 97
```

14015

30

■ (a)

```
(* no women *) Binomial[9, 5]
```

126

```
(* all possible committees *) Binomial[16, 5]
```

4368

```
(* difference *)
```

```
Binomial[16, 5] - Binomial[9, 5]
```

4242

```
(* or count all *)
```

```
(* exactly one woman, exactly two and so on *)
Table[Binomial[7, k] * Binomial[9, 5 - k], {k, 1, 6}]
```

{882, 1764, 1260, 315, 21, 0}

```
Sum[Binomial[7, k] * Binomial[9, 5 - k], {k, 1, 5}]
```

4242

■ (b)

```
(* modify the last sum *)
```

```
Sum[Binomial[7, k] * Binomial[9, 5 - k], {k, 1, 4}]
```

4221

```
(* or subtract all women committees *)

4242 - Binomial[7, 5]

4221
```

31

```
(* total number of six letter words *)

26^6

308915776

(* a *) 6 5 21^5

122523030

(* b *) Binomial[6, 2] 5^2 21^4

72930375

(* c *) 26^6 - 21^6 (* total - no vowels *)

223149655

(* d *) 26^6 - (21^6 + 6 5 21^5) (* total - no vowels - ex one *)

100626625
```

32

```
(* total number of six letter words *)

26^6

308915776
```

■ (a)

```
(* a *) (* calculate no a *) 26^6 - 25^6

64775151

(* or exactly one a + exactly two a + ... *)

Sum[Binomial[6, k] 25^(6 - k), {k, 1, 6}]

64775151
```

```
(* this is in fact a binomial theorem for (25 +1)^6 *)
```

(b)

```
(* b *) (* calcualate no a no b *)  24^6
```

191102976

```
(* this is  a or b *) 26^6 - 24^6
```

117812800

```
(* a and b = a + b - (a or b) *) 2 * (26^6 - 25^6) - (26^6 - 24^6)
```

11737502

(c)

```
25 24 23 22 21
```

6375600

(d)

```
(* all letters distinct *)
```

```
26 25 24 23 22 21
```

165765600

```
(* half of them have a to the left of b *)
```

```
26 25 24 23 22 21 / 2
```

82882800

33

```
(* committees with 3 men 3 women *)
```

```
Binomial[10, 3] * Binomial[15, 3]
```

54600

34

```
(* total number of committees of 6 *)

Binomial[10 + 15, 6]

177100

(* committees with 6 men 0 women *)

Binomial[10, 6] * Binomial[15, 0]

210

(* committees with 5 men 1 women *)

Binomial[10, 5] * Binomial[15, 1]

3780

(* committees with 4 men 2 women *)

Binomial[10, 4] * Binomial[15, 2]

22050

(* committees with 3 men 3 women *)

Binomial[10, 3] * Binomial[15, 3]

54600

(* committees with 2 men 4 women *)

Binomial[10, 2] * Binomial[15, 4]

61425

(* committees with 1 men 5 women *)

Binomial[10, 1] * Binomial[15, 5]

30030

(* committees with 0 men 6 women *)

Binomial[10, 0] * Binomial[15, 6]

5005

(* verify the sum *)

Sum[Binomial[10, k] * Binomial[15, 6 - k], {k, 0, 6}]

177100
```

```
Binomial[10 + 15, 6]
```

177100

```
(* the answer 4 women or 5 women or 6 women *)
Sum[Binomial[10, k] * Binomial[15, 6 - k], {k, 0, 2}]
```

96460

35

```
Binomial[10, 8]
```

45

36

```
Binomial[14, 5]
```

2002

37

```
2^10 - 2 (1 + 10 + Binomial[10, 2])
```

912

38

```
Binomial[45, 3] + Binomial[57, 4] + Binomial[69, 5]
```

11647713

How many possibilities are lost with the restrictions?

```
Binomial[45 + 57 + 69, 3 + 4 + 5] - (Binomial[45, 3] + Binomial[57, 4] + Binomial[69, 5])
```

879234828403848567

39

```
26 * 25 * 24 * 10 * 9 * 8
```

```
11232000
```

40

```
4 !
```

```
24
```

```
{{a, b, c, d}, {b, c, d, a}, {c, d, a, b}, {d, a, b, c}}
```

```
{{a, b, d, c}, {b, d, c, a}, {d, c, a, b}, {c, a, b, d}}
```

```
6 ! / 6
```

```
120
```

41

```
(* all tie *) 1 +
 (* two tie *) Binomial[3, 2] +
 (* clear winer, ties for 2 nd and 3 rd *) Binomial[3, 1] ( Binomial[2, 2] ) +
 (* no ties *) 3 !
```

```
13
```

42

```
(* all tie *) 1 +
 (* three tie *) Binomial[4, 3] +
 (* two tie first *) Binomial[4, 2] (1 + 2) +
 (* clear winer, ties for 2 nd and 3 rd *)
 Binomial[4, 1] ( Binomial[3, 3] + Binomial[3, 2] + Binomial[3, 1]) +
 (* no ties *) 4 !
```

```
75
```

43

```
(* all gold *) Binomial[6, 6] + Binomial[6, 5] + Binomial[6, 4] + Binomial[6, 3] +
 (* two gold, ties for silver *)
 Binomial[6, 2] * (1 + Binomial[4, 3] + Binomial[4, 2] + Binomial[4, 1]) +
 (* one gold, mult silver *)
 Binomial[6, 1] * (Binomial[5, 5] + Binomial[5, 4] + Binomial[5, 3] + Binomial[5, 2]) +
 (* one gold, one silver, mult bronze *) Binomial[6, 1] * Binomial[5, 1] *
  (Binomial[4, 4] + Binomial[4, 3] + Binomial[4, 2] + Binomial[4, 1])
```

```
873
```