## - Math 224 <br> Multivariable Calculus <br> Chapter 14

## Section 14.1 Partial derivative

- Pay attention to the notation: $f_{x}(x, y), f_{y}(x, y), \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and several other forms
- Visualizing partial derivatives on a graph
- Estimating partial derivatives using difference quotients
- Estimating partial derivatives from a contour diagram
- Using units to interpret partial derivatives

Section 14.1, Exercises and Problems: 1, 2, 4, 5, 6, 8, 9-12, 16, 17, 20-23, 30

## Section 14.2 Computing partial derivatives algebraically

- Review all derivative rules from Math 124
- What partial derivatives mean in practical terms?

Section 14.2, Exercises and Problems: 1-35 (do as many as you can), 36, 37, 38, 40, 42, 43, 44, 45

## Section 14.3 Local linearity and the differential

- Let $z=f(x, y)$ be a function of two variables. Let $(a, b)$ be a point in the domain of $f$. Then the point $(a, b, f(a, b))$ is on the graph of $f$. The graph of $f$ is the surface $z=f(x, y)$. The equation of the tangent plane to the surface $z=f(x, y)$ at the point $(a, b, f(a, b))$ is

$$
z=f(a, b)+f_{x}(a, b)(x-a)+f_{y}(a, b)(y-b)
$$

- The tangent plane approximation to $f(x, y)$ near $(a, b)$ - the local linearization
- The differential

Section 14.3, Exercises and Problems: $1-12,17,18,20,21,22,23,24,30$
Section 14.4 Gradients and the directional derivative in the plane
Let $z=f(x, y)$ be a differentiable function of two variables. Let $(a, b)$ be a point in the domain of $f$.

- The gradient vector of $f$ at the point $(a, b)$ is

$$
(\overrightarrow{\operatorname{grad}} f)(a, b)=(\vec{\nabla} f)(a, b)=f_{x}(a, b) \vec{i}+f_{y}(a, b) \vec{j}
$$

- The directional derivative of $f$ at the point $(a, b)$ in the direction of the unit vector $\vec{u}=$ $u_{1} \vec{i}+u_{2} \vec{j}$ is

$$
f_{\vec{u}}(a, b)=((\vec{\nabla} f)(a, b)) \cdot \vec{u}=f_{x}(a, b) u_{1}+f_{y}(a, b) u_{2}
$$

Section 14.4, Exercises and Problems: 1-43 (do as many as you can), 46, 48, 53-57, 63, 65-70, 73, 76, 78, 80

Section 14.5 Gradients and the directional derivative in space
Let $w=f(x, y, z)$ be a differentiable function of three variables. Let $(a, b, c)$ be a point in the domain of $f$.

- The gradient vector of $f$ at the point $(a, b, c)$ is

$$
(\overrightarrow{\operatorname{grad}} f)(a, b, c)=(\vec{\nabla} f)(a, b, c)=f_{x}(a, b, c) \vec{i}+f_{y}(a, b, c) \vec{j}+f_{z}(a, b, c) \vec{k}
$$

- The directional derivative of $f$ at the point $(a, b, c)$ in the direction of the unit vector $\vec{u}=$ $u_{1} \vec{i}+u_{2} \vec{j}+u_{3} \vec{k}$ is

$$
f_{\vec{u}}(a, b, c)=((\vec{\nabla} f)(a, b, c)) \cdot \vec{u}=f_{x}(a, b, c) u_{1}+f_{y}(a, b, c) u_{2}+f_{z}(a, b, c) u_{3}
$$

Section 14.5, Exercises and Problems: 1-40 (do as many as you can; choose various types), 41, 42, 44, 46, 48, 49, 51, 54, 55, 58.

Section 14.6 The chain rule
Section 14.6, Exercises and Problems: $1-14,16,18,20,27,29,30,31$

## Section 14.7 Second order partial derivatives

Section 14.7, Exercises and Problems: 1-30 (do most), 32, 33, 35, 36, 40, 41, 44, 45.

## Section 14.8 Differentiability

Section 14.8, Exercises and Problems: 2, 3, 4, 9, 10, 12, 17, 20
Chapter 14, Review Exercises and Problems: 45, 48, 49, 54, 55, 57, 58, 59, 102

