# Math 224 Multivariable Calculus

# Section 14.1 Partial derivative

- Visualizing partial derivatives on a graph
- Estimating partial derivatives using difference quotients
- Estimating partial derivatives from a contour diagram
- Using units to interpret partial derivatives

Section 14.1, Exercises and Problems: 1, 2, 4, 5, 6, 8, 9 - 12, 16, 17, 20 - 23, 30

### Section 14.2 Computing partial derivatives algebraically

- Review all derivative rules from Math 124
- What partial derivatives mean in practical terms?

Section 14.2, Exercises and Problems: 1 - 35 (do as many as you can), 36, 37, 38, 40, 42, 43, 44, 45

## Section 14.3 Local linearity and the differential

• Let z = f(x, y) be a function of two variables. Let (a, b) be a point in the domain of f. Then the point (a, b, f(a, b)) is on the graph of f. The graph of f is the surface z = f(x, y). The equation of the tangent plane to the surface z = f(x, y) at the point (a, b, f(a, b)) is

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b)$$

- The tangent plane approximation to f(x, y) near (a, b) the local linearization
- The differential

Section 14.3, Exercises and Problems: 1 - 12, 17, 18, 20, 21, 22, 23, 24, 30

#### Section 14.4 Gradients and the directional derivative in the plane

Let z = f(x, y) be a differentiable function of two variables. Let (a, b) be a point in the domain of f.

• The gradient vector of f at the point (a, b) is

$$\left(\overrightarrow{\operatorname{grad}} f\right)(a,b) = \left(\overrightarrow{\nabla} f\right)(a,b) = f_x(a,b)\overrightarrow{i} + f_y(a,b)\overrightarrow{j}$$

• The directional derivative of f at the point (a, b) in the direction of the unit vector  $\vec{u} = u_1 \vec{i} + u_2 \vec{j}$  is

$$f_{\overrightarrow{u}}(a,b) = \left(\left(\overrightarrow{\nabla}f\right)(a,b)\right) \cdot \overrightarrow{u} = f_x(a,b)u_1 + f_y(a,b)u_2$$

# Section 14.4, Exercises and Problems: 1 - 43 (do as many as you can), 46, 48, 53 - 57, 63, 65 - 70, 73, 76, 78, 80

### Section 14.5 Gradients and the directional derivative in space

Let w = f(x, y, z) be a differentiable function of three variables. Let (a, b, c) be a point in the domain of f.

• The gradient vector of f at the point (a, b, c) is

$$\left(\overrightarrow{\operatorname{grad}} f\right)(a,b,c) = \left(\overrightarrow{\nabla} f\right)(a,b,c) = f_x(a,b,c)\overrightarrow{i} + f_y(a,b,c)\overrightarrow{j} + f_z(a,b,c)\overrightarrow{k}$$

• The directional derivative of f at the point (a, b, c) in the direction of the unit vector  $\vec{u} = u_1 \vec{i} + u_2 \vec{j} + u_3 \vec{k}$  is

$$f_{\overrightarrow{u}}(a,b,c) = \left(\left(\overrightarrow{\nabla}f\right)(a,b,c)\right) \cdot \overrightarrow{u} = f_x(a,b,c)u_1 + f_y(a,b,c)u_2 + f_z(a,b,c)u_3$$

- Section 14.5, Exercises and Problems: 1 40 (do as many as you can; choose various types), 41, 42, 44, 46, 48, 49, 51, 54, 55, 58.
- Section 14.6 The chain rule
- Section 14.6, Exercises and Problems: 1 14, 16, 18, 20, 27, 29, 30, 31
- Section 14.7 Second order partial derivatives
- Section 14.7, Exercises and Problems: 1 30 (do most), 32, 33, 35, 36, 40, 41, 44, 45.
- Section 14.8 Differentiability
- Section 14.8, Exercises and Problems: 2, 3, 4, 9, 10, 12, 17, 20
- Chapter 14, Review Exercises and Problems: 45, 48, 49, 54, 55, 57, 58, 59, 102