MATH 224 Examination 1
January 20, 2012

Name Key

GIVE DETAILED EXPLANATIONS FOR YOUR ANSWERS. THERE ARE FOUR PROBLEMS. EACH IS WORTH 25 POINTS.

1. The graph in Figure 1 shows the contour line at level 0 of a plane. This plane also passes through the point $(-1, 1, 1)$.
   (a) Find the equation $z = ax + by + c$ of this plane.
   (b) Determine the intercepts of this plane with the coordinate axes x, y and z.

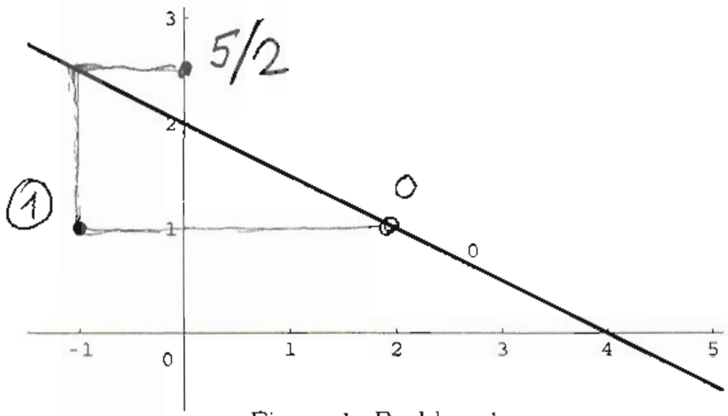

Figure 1: Problem 1

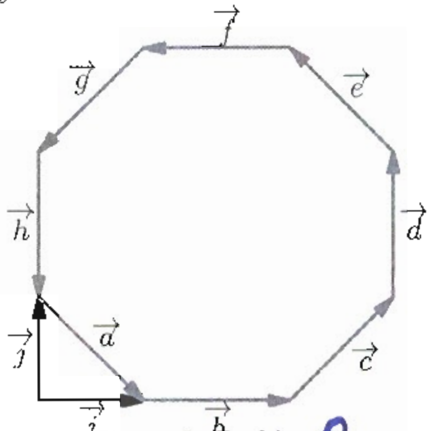

Figure 2: Problem 2

2. There are 10 vectors in Figure 2. Two black vectors are the coordinate vectors $\vec{i}$ and $\vec{j}$. The eight gray vectors form a regular octagon. Express all eight gray vectors in terms of $\vec{i}$ and $\vec{j}$.

3. Consider the following two functions $f(x, y) = \dfrac{y}{\sqrt{x^2 + y^2}}$ and $g(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}}$. The graphs of these functions are given in Figures 3 and 4.
   (a) One of these functions does not have a limit as $(x, y) \to (0, 0)$. Determine which one and justify your claim by using lines $y = mx$.
   (b) One of these functions does have a limit as $(x, y) \to (0, 0)$. Determine which one and based on its graph given below state what is the limit.

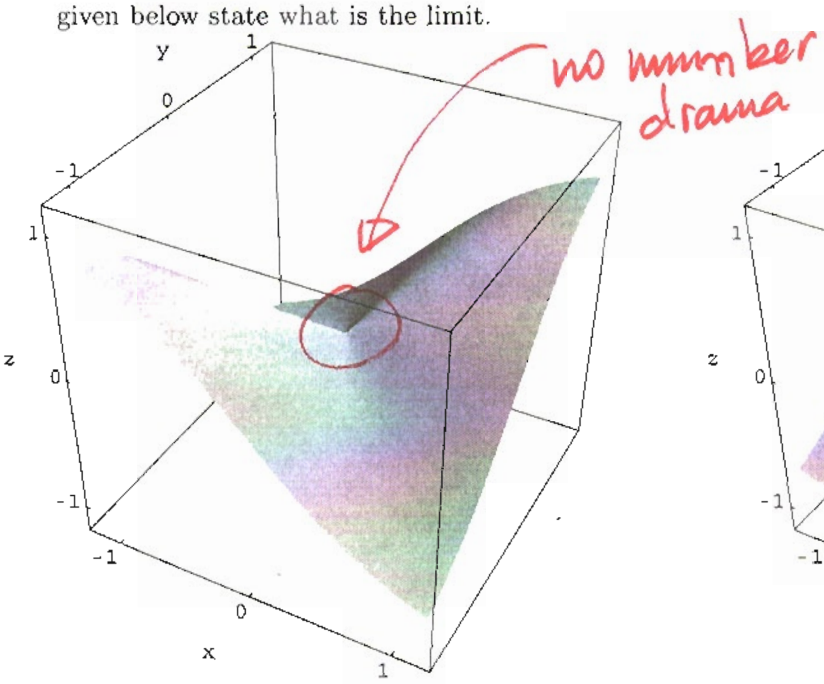

Figure 3: $f(x, y)$ or $g(x, y)$

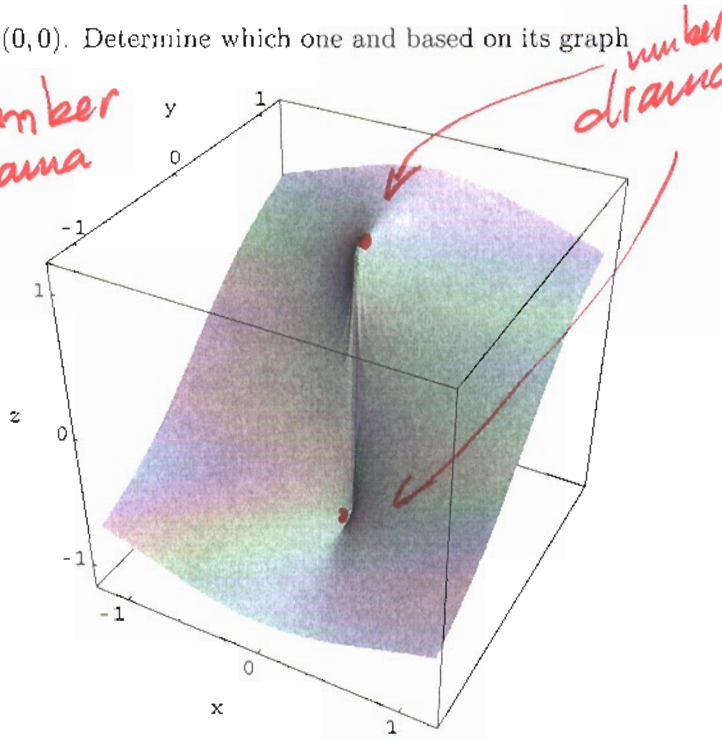

Figure 4: $f(x, y)$ or $g(x, y)$

OVER

4. The table below gives eight functions of three variables. Each function is labeled by a capital letter from A to H.

(A) $\cancel{w = x^2 - z^2}$	(B) $w = x^2 - y^2 - z^2$	(C) $w = x^2 - z$	(D) $w = x^2 + y^2 - z^2$
(E) $w = -x^2 + y^2 + z^2$	(F) $w = x^2 - y^2 - z$	(G) $\cancel{w = x^2 + z^2}$	(H) $w = x^2 + y^2 - z$

For **six** out of the **eight** functions given above I created the level surfaces corresponding to $w = 1$. These six level surfaces are pictured below.

Match six of the above functions with their level surfaces pictured below. Mark the box containing the level surface by the letter corresponding to the function whose level surface with $w = 1$ is given in the picture.

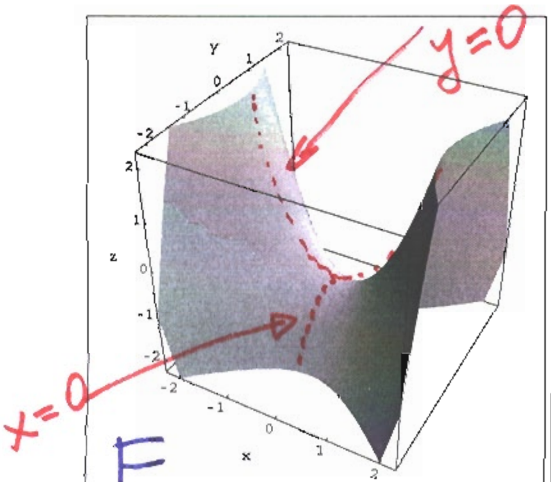

Figure 5

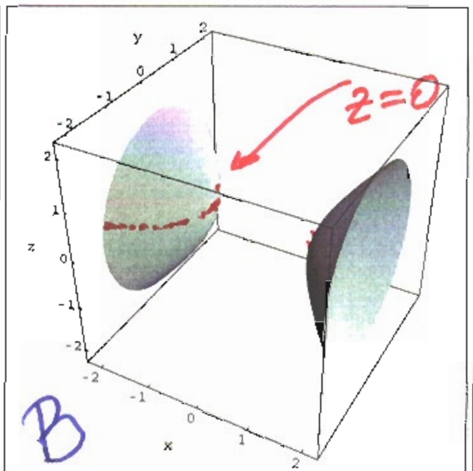

Figure 6

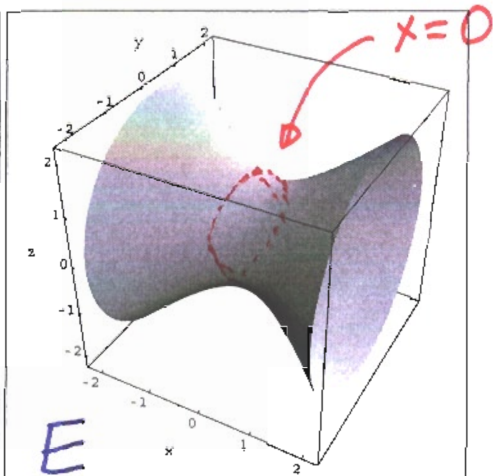

Figure 7

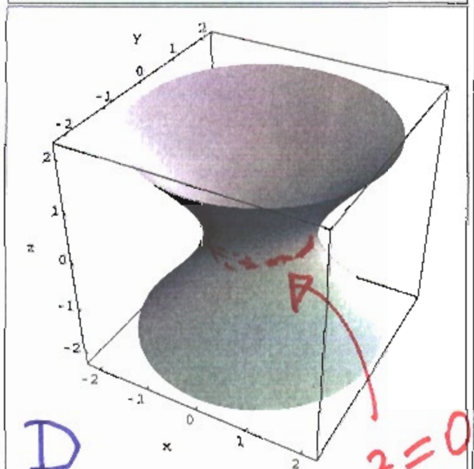

Figure 8

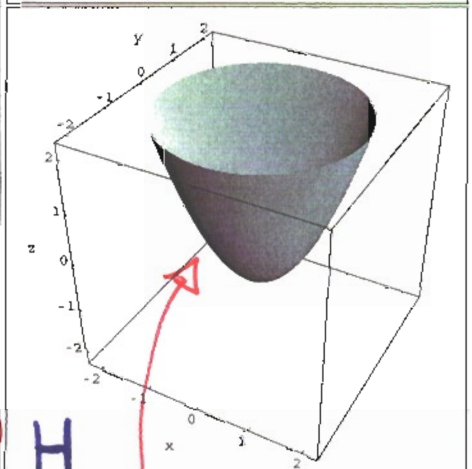

Figure 9

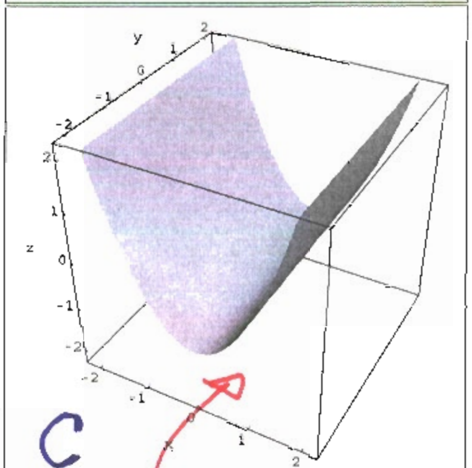

Figure 10

$z = x^2 + y^2 - 1$ $z = x^2 - 1$

1 a $m = \frac{-1}{3} = -\frac{1}{3}$

$n = \frac{-1}{3/2} = -\frac{2}{3}$

see the picture

$z = -\frac{1}{3}x - \frac{2}{3}y + c$

$1 = -\frac{1}{3}(-1) - \frac{2}{3}\cdot 1 + c$

$\boxed{\frac{4}{3} = c}$ $\boxed{z = -\frac{1}{3}x - \frac{2}{3}y + \frac{4}{3}}$

1b for x-axes

$y=0, z=0$

so $x = 4$

the point is $\boxed{(0,0,4)}$

for y axes

$x=0, z=0$ so

$y = 2$

the point is $\boxed{(0,2,0)}$

z-axes

$\boxed{(0,0,4/3)}$

2

$\vec{a} = \vec{i} - \vec{j}$

$\vec{b} = \sqrt{2}\,\vec{i}$

$\vec{c} = \vec{i} + \vec{j}$

$\vec{d} = \sqrt{2}\,\vec{j}$

$\vec{e} = \vec{j} - \vec{i}$

$\vec{f} = -\sqrt{2}\,\vec{i}$

$\vec{g} = -\vec{i} - \vec{j}$

$\vec{h} = -\sqrt{2}\,\vec{j}$

③ ⓐ The function f does not have a limit as $(x,y) \to (0,0)$

$\boxed{2}$

Consider the line $y=0$,

then $f(x,0) = 0$ for all $x \neq 0$

Consider the line $x=0$

Then $f(0,y) = \frac{y}{\sqrt{y^2}} = \frac{y}{|y|}$ for $y \neq 0$

For $y>0$ $f(0,y) = 1$

for $y<0$ $f(0,y) = -1$

Since f takes values; $1, -1, 0$ near $(0,0)$. $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist

(3b) Based on Figure 3 $\boxed{3}$

which represents the function $g(x,y)$ I see that $\lim_{(x,y)\to(0,0)} g(x,y) = 0$.

This can be confirmed using lines $y = mx$ and calculating

$$g(x,mx) = \frac{mx^2}{\sqrt{x^2+m^2x^2}} = \frac{mx^2}{|x|\sqrt{1+m^2}} =$$

$$= \underbrace{|x| \frac{m}{\sqrt{1+m^2}}}$$

This quantity is small whenever the point (x,y) is close to $(0,0)$.