MATH 226

Assignment 4
February 23, 2012

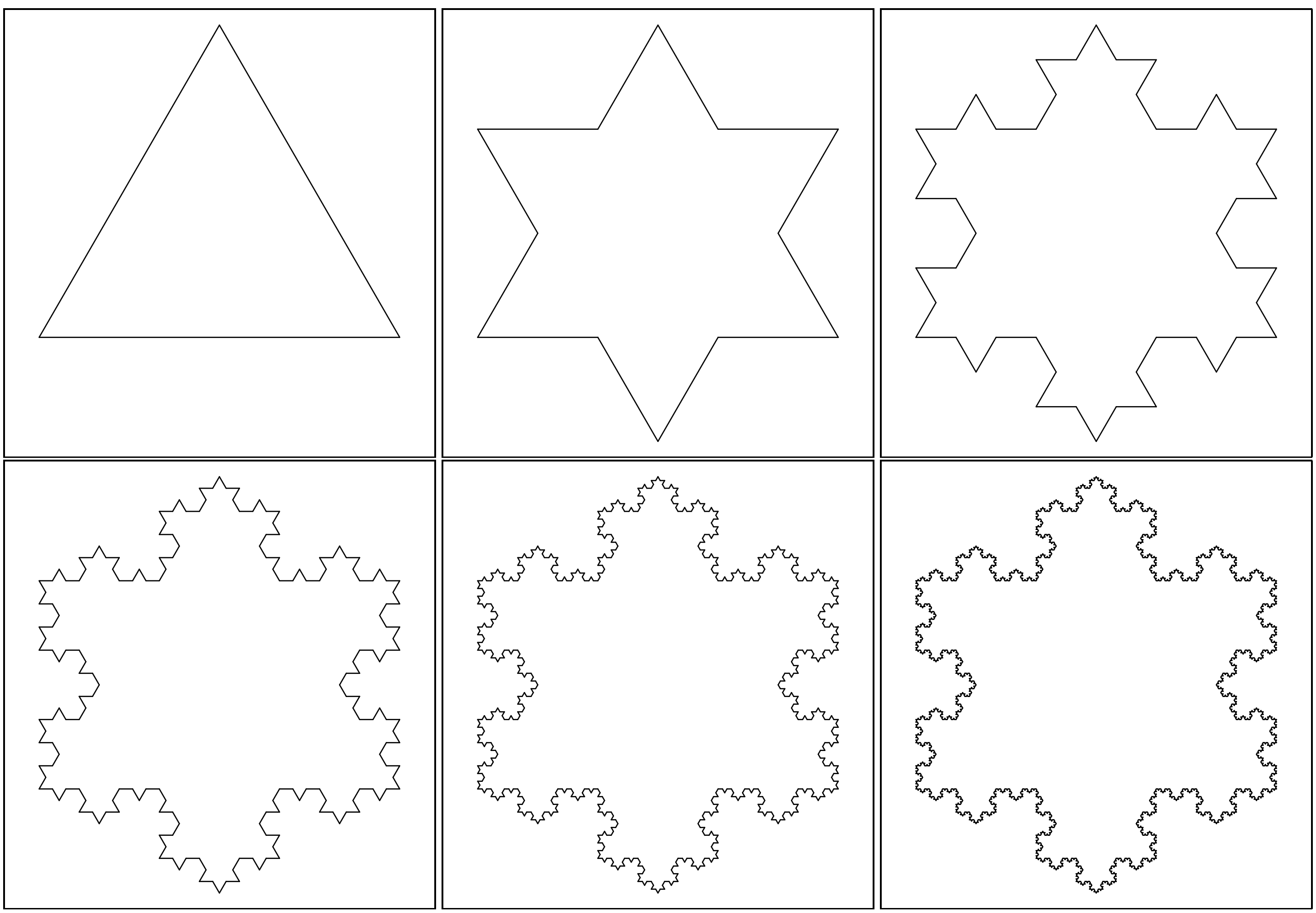
Name ______________________________

In this assignment you are asked to study two famous mathematical objects and two much less famous objects. What unifies them is that the tool to study them is the same: geometric series. The famous objects the Koch snowflake and the Sierpinski carpet. Since these are famous objects there is a lot of information about them on the internet. So, the answers to the questions that are aske here are probably available. But, please present your own work here. What we learned about sequnces and series should be sufficient to answer these questions.

Exercise 1. The Koch snowflake is a figure that is constructed in the following way: The Koch snowflake at step 0, denoted by K_0, is an equilateral triangle with sides of length 1. Then, on each step we break every side of the figure into three equal segments, on every middle segment we build an equilateral triangle (facing outwards from our figure) and finally we throw the middle segments away. The Koch snowflakes $K_0, K_1, K_2, K_3, K_4, K_5$ are given in the figures below.

(a) Calculate the length L_n of the Koch snowflake K_n. What is the limit of L_n as $n \to +\infty$?

(b) Calculate the area A_n enclosed by the Koch snowflake K_n. What is the limit of A_n as $n \to +\infty$?



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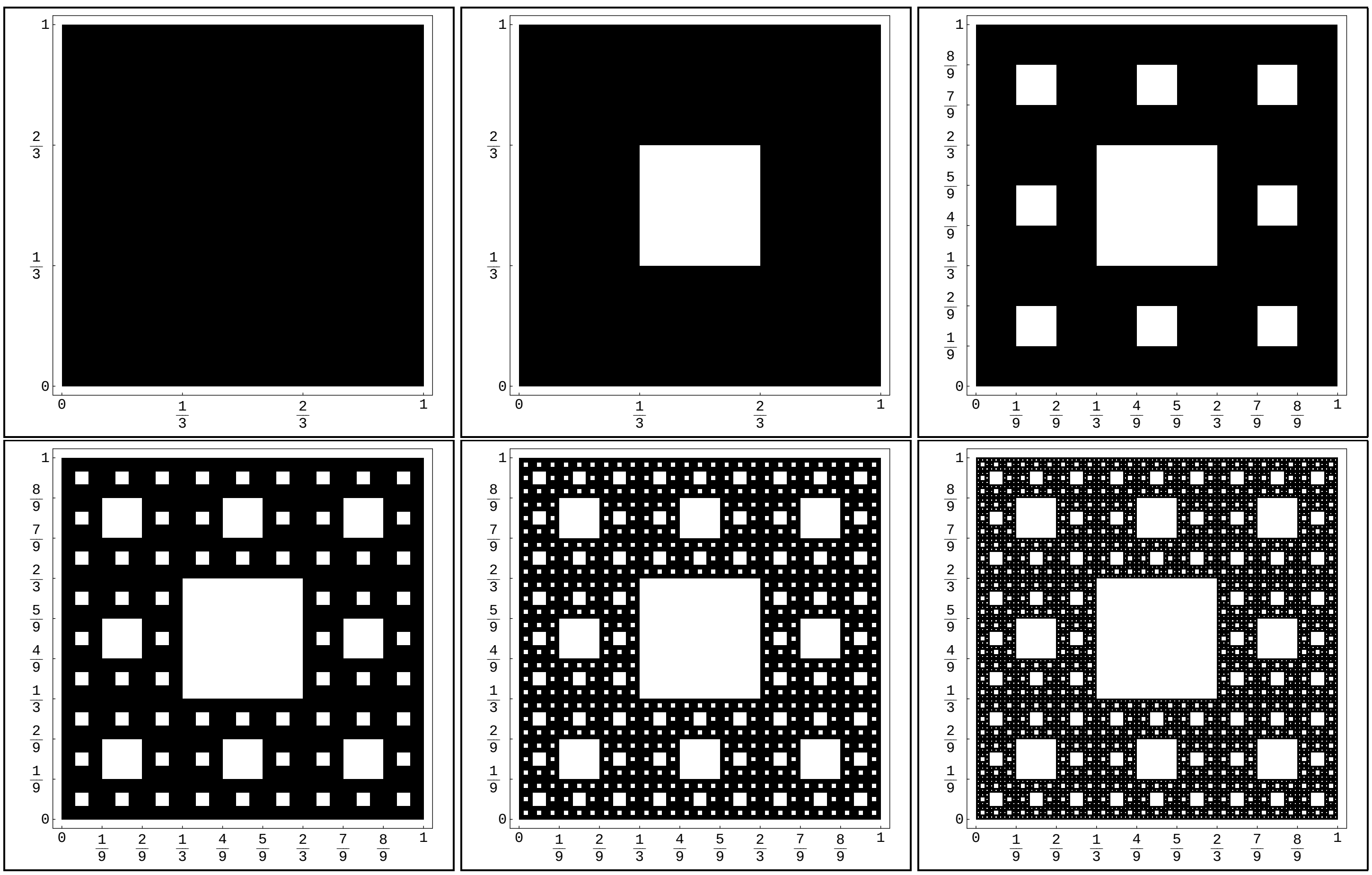
Exercise 2. The Sierpinski carpet is a figure that is constructed in the following way: The Sierpinski carpet at step 0, denoted by S_0, is a unit square. At step 1 the square is cut into 9 congruent subsquares in a 3-by-3 grid, and the central subsquare is removed. The remaining 8 subsquares form the figure is S_1. The same procedure is then applied recursively to each of the 8 subsquares to obtain S_2. Continuing this procedure recursively we obtain S_n for each $n \in \mathbb{N}$. The Sierpinski carpets $S_0, S_1, S_2, S_3, S_4, S_5$ are given in the figures below.

(a) Calculate the area A_n of the Sierpinski carpet S_n. What is the limit of A_n as $n \to +\infty$?

(b) Are there points that are in all Sierpinski carpets S_n? That is, are there points P such that $P \in S_n$ for all $n \in \mathbb{N}$? Identify some of these points.



Exercise 3. Some of us are lucky enough to have the initials which are hexadecimal digits. So, we might be curious which number has hexadecimal expansion $0.BCBCBCBCBC\ldots$. For the full entertainment value, represent this number as a fraction of two natural numbers in hexadecimal numeral system.

Exercise 4. You might think that the previous exercise is somewhat egotistical. If that is the case, or in any case, repeat the previous exercise for the number $0.ABCABCABCABCABC\ldots$. But, in this case, please do not forget to simplify the fraction.