Few basic properties of inequalities

Transitivity:

For $a,\ b,\ c \in \mathbb{R}$ we have that $a \le b$ and $b \le c$ implies $a \le c$

Respect for multiplication:

For $a,\ b,\ c \in \mathbb{R}$ we have that $a \le b$ and $0 \le c$ implies $a\,c \le b\,c$

Respect for the reciprocal:

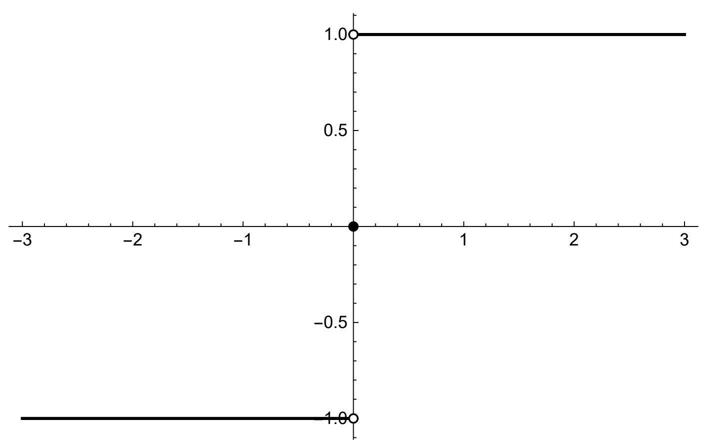
For $a \in \mathbb{R}$ we have that $0 < a$ implies $0 < \frac{1}{a}$

Review of functions

The sign function

sign $x > 0$ then Sign[x] = 1, x =0 then Sign[x] = Sign[0] = 0, if x < 0 then Sign[x] = -1

In[ ]:=
```
Plot[Sign[x], {x, -3, 3}, PlotStyle → {Black}, Epilog → {{PointSize[0.015], Point[{0, 0}]},
   {PointSize[0.015], Point[{0, 1}], PointSize[0.01], White, Point[{0, 1}]},
   {PointSize[0.015], Point[{0, -1}], PointSize[0.01], White, Point[{0, -1}]}}]
```

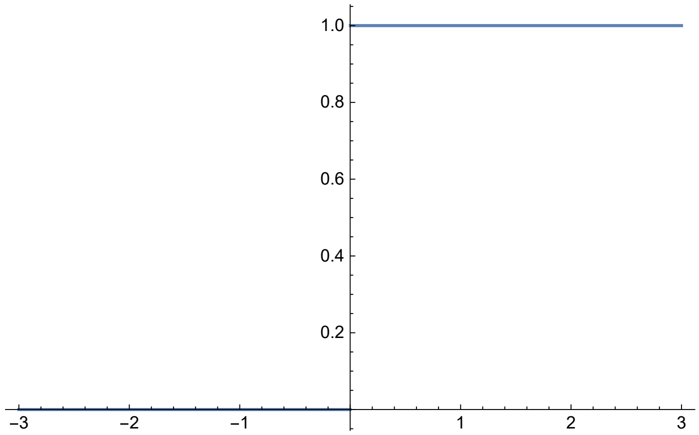
Out[ ]=


The unit step function

unit step function

In[∘]:=
```
Plot[UnitStep[x], {x, -3, 3}]
```

Out[∘]=

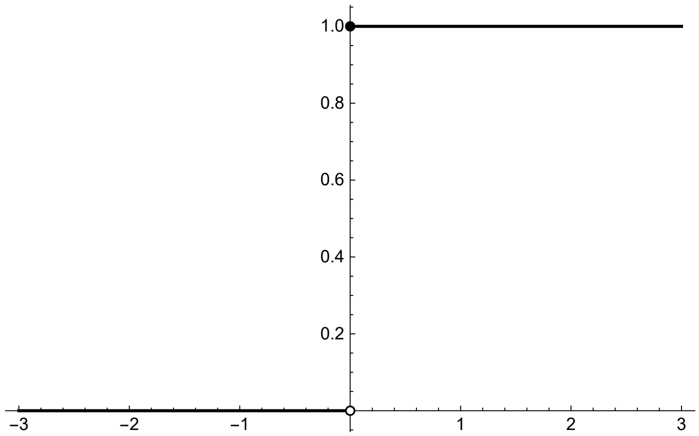


In[∘]:=
```
UnitStep[0]
```

Out[∘]= 1

In[∘]:=
```
Plot[UnitStep[x], {x, -3, 3}, PlotStyle → {Black},
 Epilog → {{PointSize[0.015], Point[{0, 1}]},
   {PointSize[0.015], Point[{0, 0}], PointSize[0.01], White, Point[{0, 0}]}}]
```

Out[∘]=



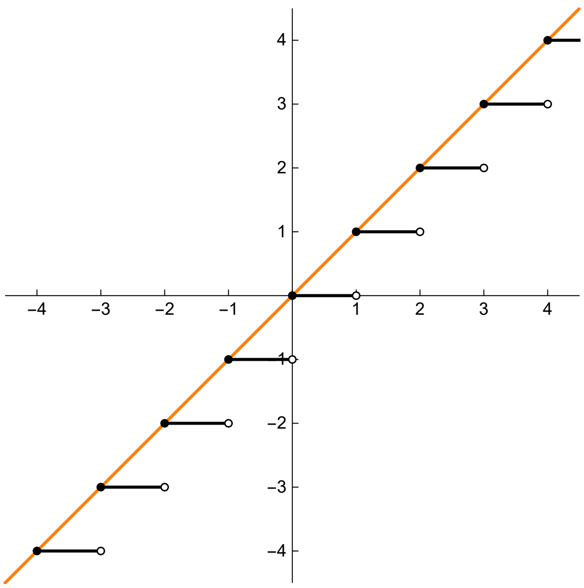
The formal definition of the unit step function is

$x \in \mathbb{R}$ if $x < 0$ then us(x) = 0, if $x \geq 0$ then us(x) = 1

The floor function

In[25]:=
```
Plot[{x, Floor[x]}, {x, -8, 8}, PlotStyle → {{RGBColor[1, 0.5, 0]}, {Black}},
 Epilog → {{PointSize[0.015], Black, Table[Point[{k, k}], {k, -10, 10}]},
   {PointSize[0.015], Table[Point[{k + 1, k}], {k, -10, 10}],
    PointSize[0.01], White, Table[Point[{k + 1, k}], {k, -10, 10}]}},
 AspectRatio → Automatic, PlotRange → {{-4.5, 4.5}, {-4.5, 4.5}},
 Ticks → {Range[-5, 5], Range[-5, 5]}, ImageSize → 300]
```

Out[25]=



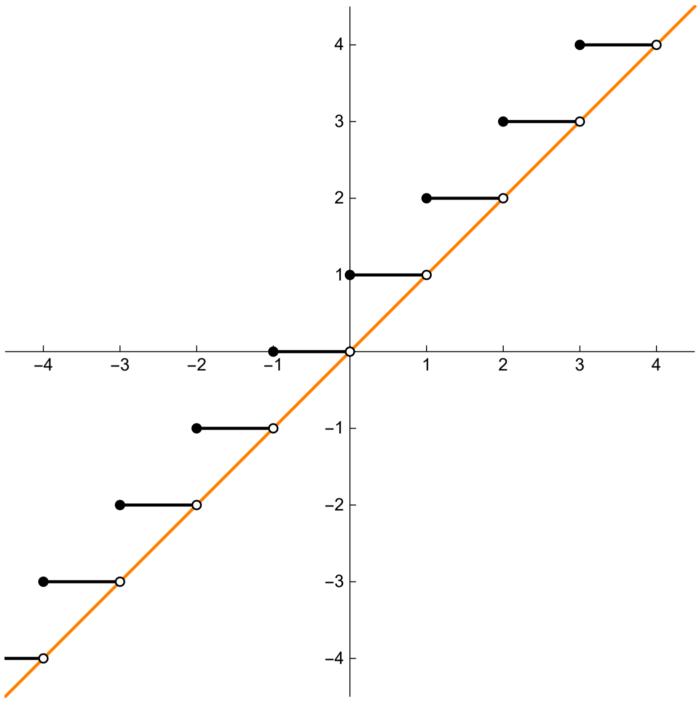
Formal definition of the floor function!

For $x \in \mathbb{R}$ we define: Floor[x] = $\lfloor x \rfloor = \max\{k \in \mathbb{Z} : k \leq x\}$

The ceiling function

In[26]:=
```
Plot[{x, Ceiling[x]}, {x, -8, 8}, PlotStyle → {{RGBColor[1, 0.5, 0]}, {Black}},
 Epilog → {{PointSize[0.015], Black, Table[Point[{k, k + 1}], {k, -10, 10}]},
   {PointSize[0.015], Table[Point[{k, k}], {k, -10, 10}], PointSize[0.01],
    White, Table[Point[{k, k}], {k, -10, 10}]}}, AspectRatio → Automatic,
 PlotRange → {{-4.5, 4.5}, {-4.5, 4.5}}, Ticks → {Range[-5, 5], Range[-5, 5]}]
```

Out[26]=



Formal definition of the ceiling function!

For $x \in \mathbb{R}$ we define: Ceiling[x] = $\lceil x \rceil = \min\{k \in \mathbb{Z} : k \geq x\}$

In[ ]:=
```
Ceiling[Pi]
```

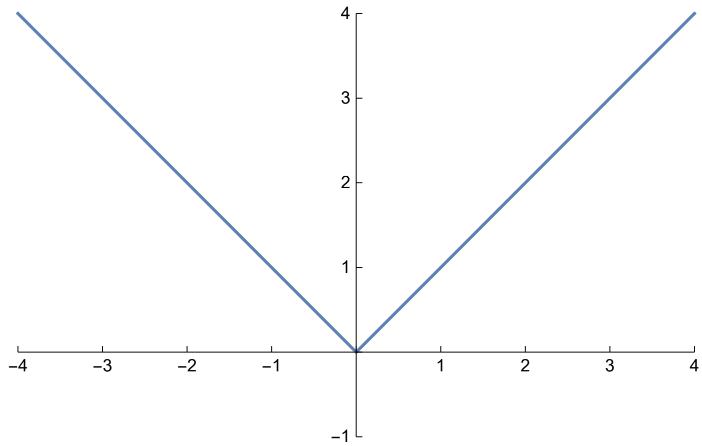
Out[ ]= 4

The absolute value function

abs

In[27]:=
```
Plot[{Abs[x]}, {x, -4, 4}, AspectRatio → Automatic,
 PlotRange → {{-4, 4}, {-1, 4}}, Ticks → {Range[-5, 5], Range[-5, 5]}]
```

Out[27]=



Think of the real number line as a special highway. How far is π from e? $\pi - e$

Think of the real number line as a special highway. How far is π^e from e^π? $| \pi^e - e^\pi |$

The definition of the absolute value function. Let $x \in \mathbb{R}$. Then if $x < 0$ then $\text{abs}(x) = -x$, if $x \geq 0$ then $\text{abs}(x) = x$.

Exercise: Prove $\text{abs}(x) = x \, \text{sign}(x)$ for all $x \in \mathbb{R}$.

Triangle inequality