$\forall a, b \in \mathbb{R} \quad |a+b| \leq |a|+|b|$ unknown needs proof

G1 $\boxed{\forall x \in \mathbb{R} \quad x \leq |x| \quad -x \leq |x|}$ known! → Green

Proof. Let $a, b \in \mathbb{R}$ be arbitrary. $a+b \in \mathbb{R}$.

Case 1 $a+b \geq 0$ Case 2 $a+b < 0$

Case 1 $a+b \geq 0 \Rightarrow |a+b| = a+b$, def abs

G1 $a \leq |a|$, $b \leq |b|$ $\Rightarrow \boxed{a+b \leq |a|+|b|}$ respect

$\Downarrow$ $|a+b| \leq |a|+|b|$

Case 2 $a+b < 0$ alg.

$|a+b| \underset{\text{def abs}}{=} -(a+b) = -a+(-b) \leq |a|+|b|$

$= (-a)+(-b)$ G1 $-a \leq |a|$, $-b \leq |b|$ $\Rightarrow -a-b \leq |a|+|b|$ respect

$\boxed{|a+b| \leq |a|+|b|}$ GREEN ✓

$\forall a,b \in \mathbb{R}$ $\quad |a+b| \leq |a|+|b|$ unknown needs proof

G1 $\boxed{\forall x \in \mathbb{R} \quad x \leq |x| \quad -x \leq |x|}$ known ! $\Rightarrow$ Green

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Case 1 $a+b \geq 0 \Rightarrow |a+b| = a+b$ G1 $a \leq |a|$, $b \leq |b|$ $\Rightarrow \boxed{a+b \leq |a|+|b|}$

def abs

$\Downarrow$ $|a+b| \leq |a|+|b|$

Case 2 $a+b<0$ alg.

$|a+b| \underset{\text{def abs}}{=} -(a+b) = -a+(-b) \leq |a|+|b|$

$(-a)+(-b)$ G1 $-a \leq |a|$, $-b \leq |b|$ respect $\Rightarrow -a-b \leq |a|+|b|$

$\boxed{|a+b| \leq |a|+|b|}$ GREEN ✓

$$\left.\begin{array}{l} a \le b \\ c \le d \end{array}\right\} \Rightarrow \underline{a+c} \le \underline{b+d}$$

<u>309</u> <u>Integers</u>

$\boxed{\forall \text{ day in Bell it rains}}$

neg. of this

<u>$\exists$ day</u> in Bell it did not.

Counter exa

$$\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \tfrac{1}{2} \rfloor$$

$\underline{\underline{\forall x \in \mathbb{R}}}$ $$\left\lfloor \frac{\lfloor 2x \rfloor}{2} \right\rfloor = \lfloor x \rfloor \;?$$

$$\forall x,y \in \mathbb{R} \quad \big|\underbrace{|x|-|y|}\big| \le |x-y|$$

G1 $\boxed{|z| = \max\{-z, z\}}$

Proof: Part 1 Do prove $\boxed{|x|-|y| \le |x-y|}$

Let $x,y \in \mathbb{R}$ be arb.

$\boxed{x = x-y+y}$ subst. TI $|a+b| \le |a|+|b|$

$\boxed{|x| \le |x-y|+|y|}$

$$\boxed{|x| \le |x-y| + |y|} \overset{\text{alg}}{\Rightarrow} \boxed{|x| - |y| \le |x-y|}$$

add $-|y|$

respect order add

TI

Part B

a

b in TI

$$y = y - x + x$$

respect OA

By TI

$$|y| \le |\underbrace{y-x}_{a} + \underbrace{x}_{b}| \le |y-x| + |x| \Rightarrow$$

$$\boxed{|z| = |-z|}$$

$$\boxed{|y| - |x| \le |y-x|}$$

$$\boxed{|y| - |z| \le |x-y|}$$

$$\boxed{|x| \le |x-y| + |y|} \overset{\text{alg}}{\Longrightarrow} \boxed{|x| - |y| \le |x-y|} \quad G3$$

add $-|y|$

respect order add

Part B

$$y = y - x + x$$

By TI $\quad |y| \le |y - x + x| \le |y-x| + |x| \Rightarrow$

respect OA

$$\boxed{|z| = |-z|}$$

$$\boxed{|y| - |x| \le |y-x|}$$

$$\boxed{|y| - |z| \le |x-y|} \quad G4$$

G3 $\boxed{|x|-|y| \le |x-y|}$ G4 $\boxed{|y|-|x| \le |x-y|}$

$$\big||x|-|y|\big| = \max\{|x|-|y|, -(|x|-|y|)\}$$

alg

$$= \max\{\underbrace{|x|-|y|}_{\le |x-y|}, \underbrace{|y|-|x|}_{\le |x-y|}\}$$

$$\le |x-y|$$