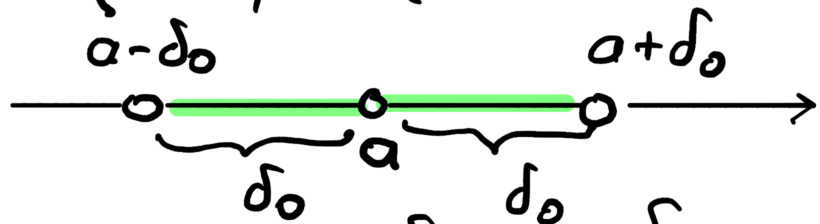
Examples of limits at a

April 21, 2020

Def of limit at a point $a \in \mathbb{R}$. Here $f: D \to \mathbb{R}$, $D \subseteq \mathbb{R}$, $L \in \mathbb{R}$

(I) $\exists\, \delta_0 > 0$ s.t. $(a-\delta_0, a) \cup (a, a+\delta_0) \subseteq D$



(II) $\forall \varepsilon > 0\ \exists\, \delta(\varepsilon) > 0$ s.t. $\delta(\varepsilon) \le \delta_0$ and

$$\underbrace{0 < |x-a|}_{\text{exludes } a} < \delta(\varepsilon) \Rightarrow |f(x) - L| < \varepsilon$$

the famous lim of this kind $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

Example $\lim\limits_{x\to 2}(3x-1) = 5$, $D=\mathbb{R}$

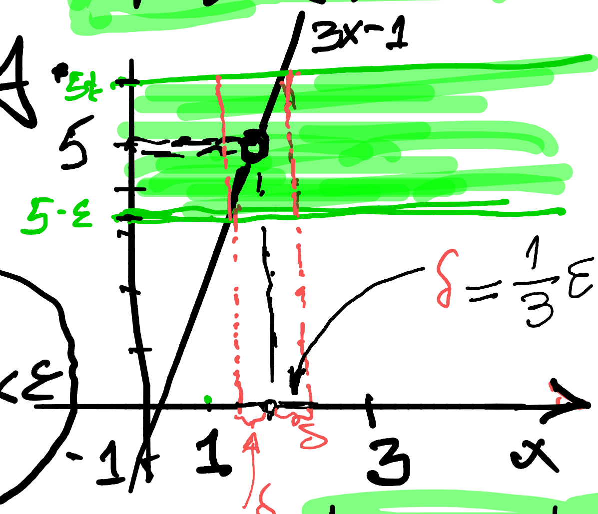
$a=2,\ L=5$

Proof. (I) $\delta_0 = 1 > 0$

Clearly $(1,2)\cup(2,3)\subseteq \mathbb{R}$

(II) Let $\varepsilon>0$ be arbitrary.

$$\delta(\varepsilon)=\min\left\{\frac{1}{3}\varepsilon, 1\right\}>0$$



Prove

$$0<|x-2|<\min\left\{\frac{1}{3}\varepsilon,1\right\} \Rightarrow |3x-1-5|<\varepsilon$$

Proof. Assume $|x-2|<\frac{1}{3}\varepsilon$

$$\Rightarrow 3|x-2|<\varepsilon \Rightarrow |3x-6|<\varepsilon \Rightarrow |3x-1-5|<\varepsilon$$

Ex $\lim_{x \to 3} x^2 = 9$

$D = \mathbb{R}$, $L = 9$

$a = 3$

Proof: (I) $\delta_0 = 1$ $\quad (2,3) \cup (3,4) \subseteq \mathbb{R}$

(II) Let $\varepsilon > 0$ be arbitrary.

$\vdots$