The first proof in Math 226

and about quantifiers

$\forall$ for all and $\exists$ exists

universal quantifier existensial quantifier

Logic Makes Sense!

p, q propositions ; compound propositions (new prop. from old)

inclusive

$\neg p$, $p \wedge q$, $p \vee q$, $p \Rightarrow q$, $p \Leftrightarrow q$

negation

$\neg(\neg p) \Leftrightarrow p$

$\neg(p \wedge q) \Leftrightarrow (\neg p) \vee (\neg q)$

$\neg(p \vee q) \Leftrightarrow (\neg p) \wedge (\neg q)$

$\neg(p \Rightarrow q) \Leftrightarrow p \wedge (\neg q)$

$\neg(p \Leftrightarrow q) \Leftrightarrow (\neg p) \oplus (\neg q)$

exclusive disjunction

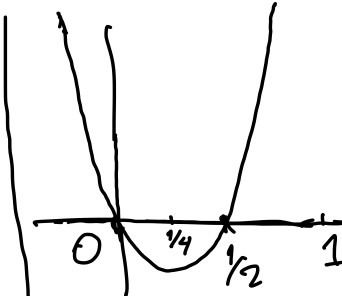
Propositional functions $Q(x)$: $2x^2 - x \geq 0$

$Q(1)$ is $2 \geq 0$ T

$Q(1/2)$ is $2 \cdot 1/4 - 1/2 = 0 \geq 0$ T

$Q(1/4)$ is $2 \cdot 1/16 - 1/4 = -1/8 \geq 0$ F

$Q(1) \wedge Q(1/2) \wedge \underbrace{Q(1/4)}_{F}$ F

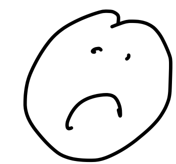


universal quantifier $\forall$ (for all)

$$\boxed{\forall x \in \mathbb{R} \quad 2x^2 - x \geq 0} \quad F$$

a proposition

$a > 0$



$a < 0$

existential quantifier $\exists$ (exists)

$$\exists x \in \mathbb{R} \quad 2x^2 - x \geq 0 \quad T$$

LaTeX
\forall
\exists

$P(x)$ any propositional function

domain is called $\underbrace{\text{UNIVERSE of DISCOURSE}}_{\text{call it } U}$

$$\boxed{\forall x \in U \quad P(x)}$$ a proposition

$$\neg\left(\forall x \in U \ P(x)\right) \iff \left(\exists x \in U \ \neg P(x)\right)$$

"conjunction of many props" "disjunction of many prop."

$$\neg\left(\exists x \in U \ P(x)\right) \iff \left(\forall x \in U \ \neg P(x)\right)$$

multiple quantifiers $Q(a,x): ax^2-x \geq 0$

$$\forall a \in \mathbb{R} \ \exists x \in \mathbb{R} \quad ax^2 - x \geq 0$$

such that

This is a proposition $\rightarrow$ True or F?

This is MATH.

Negate & consider both original & its negation

$$\exists a \in \mathbb{R} \ \forall x \in \mathbb{R} \quad ax^2 - x < 0$$

This is False

$a = -1 \quad \forall x \in \mathbb{R} \ -x^2 - x < 0$?

ORANGE is TRUE. Let $a \in \mathbb{R}$ be arbitrary. Take $x = 0$ $\quad a0^2 - 0 = 0 \geq 0$

Prop 5.1. $a, b, c \in \mathbb{R}$

$$\underbrace{\forall x \in \mathbb{R} \quad ax^2+bx+c \geq 0}_{\text{a proposition}} \implies a \geq 0$$

CONTRAPOSITIVE is easier to prove.

$$\boxed{a < 0} \implies \neg\left(\forall x \in \mathbb{R} \ ax^2+bx+c \geq 0\right)$$

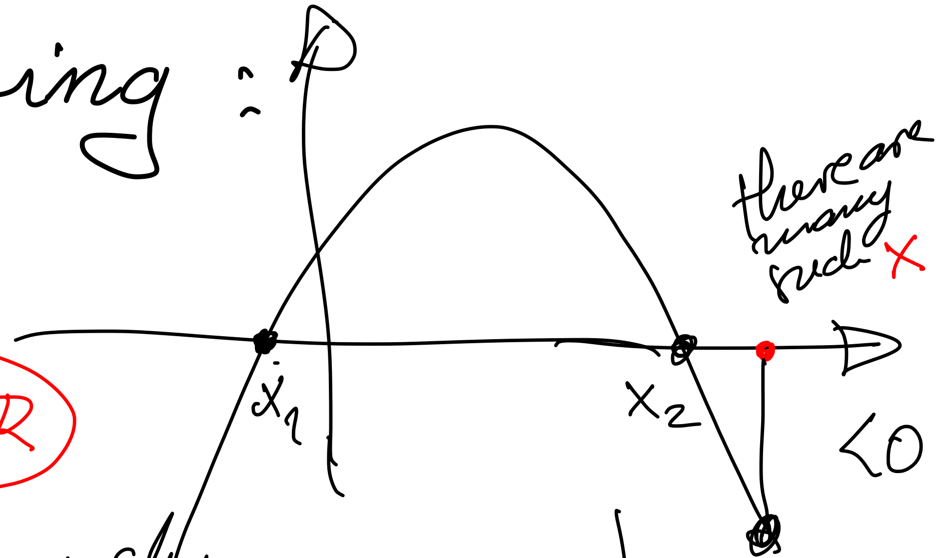
$$\boxed{\exists x \in \mathbb{R} \quad \underline{ax^2+bx+c < 0}}$$

hypothesis is always green

Sloppy thinking :

$\boxed{a<0}$

my task → $\exists x \in \mathbb{R}$



To do this rigorously we need to find a formula for x. done

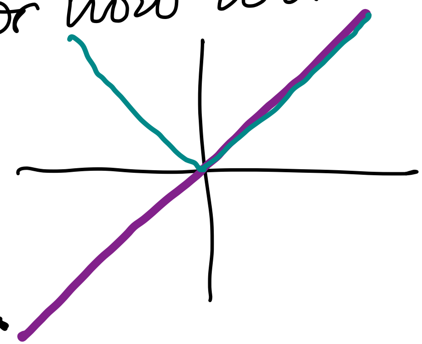
Now I ~~can~~ must be creative.

I will restrict my search to $x \in \mathbb{R}$ $\boxed{x \geq 1}$

You learned about $|\cdot|$. For all real numbers we have

$$b \le |b| \quad \text{and} \quad c \le |c|.$$

We will prove this latter. For now look at the graphs: x and $|x|$



The graphs show $x \le |x|$ for all $x \in \mathbb{R}$.

Next: You learned that the following implications are true

$$(x \ge 1) \wedge (b \le |b|) \Rightarrow bx \le |b|x$$

$$(x \ge 1) \wedge (|c| \ge 0) \Rightarrow |c| \le |c|x$$

$$(c \le |c|) \wedge (|c| \le |c|x) \Rightarrow c \le |c|x$$

The following implication is also true:

$$(bx \leq |b|x) \wedge (c \leq |c|x) \Rightarrow (ax^2+bx+c \leq ax^2+|b|x+|c|x)$$

Now we study $ax^2+(|b|+|c|)x = (ax+|b|+|c|)x$

Solve $ax+|b|+|c| = a$ for x

$$x_0 = 1 - \frac{|b|+|c|}{a} \geq 1 \text{ since } a<0$$

$$ax_0^2+|b|x_0+|c|x_0 = (ax_0+|b|+|c|)x_0 = a-|b|-|c| < 0$$

since $a<0$.

Since $x_0 \geq 1$ we have

$$ax_0^2+bx_0+c \leq ax_0^2+|b|x_0+|c|x_0 = a-|b|-|c| < 0$$

Thus $ax_0^2+bx_0+c < 0$.

thus we proved that

$a < 0 \Rightarrow \exists x \in \mathbb{R}$ such that $ax^2+bx+c<0$

this is $x_0 = 1 - \frac{|b|+|c|}{a}$

In colors:

$$x = 1 - \frac{|b|+|c|}{a}$$

red x is expressed in terms of green a, b, c.