More about

Absolute Convergence

In Problem 6 on A3,

hexadecimal numbers (geometric series)

html colors RGB # $\underbrace{\sqcup\sqcup}_{\text{red}}\underbrace{\sqcup\sqcup}_{\text{green}}\underbrace{\sqcup\sqcup}_{\text{blue}}$

red green blue

0, ..., 255

base 16

base 10 our digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

$$376 = 3 * 10^2 + 7 * 10^1 + 6 * \underbrace{10^0}_{1}$$

$$0.376 = 3 * 10^{-1} + 7 * 10^{-2} + 6 * 10^{-3}$$

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base 16 digits are 0, 1, 2, ..., 9, A, B, C, D, E, F

A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

FF in decimal $\underbrace{15*16}+15*16^0=240+15=255$

$16*16=256$ $\quad 15*16=160+80$

red in hex code is $\#\underbrace{FF}_{\text{red}}\underbrace{00}_{\text{gr}}\underbrace{00}_{\text{blue}}$ dark red $\#\underbrace{80}\underbrace{0000}$

black #000000

80_{hex} is $8*16+0*16^0=128$

Just another example: base 5: the digits 0, 1, 2, 3, 4

7_{dec} $\quad 1*5+2*5^0$

$12_{base 5}$

dec	base 5
0	0
1	1
2	2
3	3
4	4
5	10
6	11
7	12

Back to Absolute Convergence

↳ contrast to Conditional Convergence

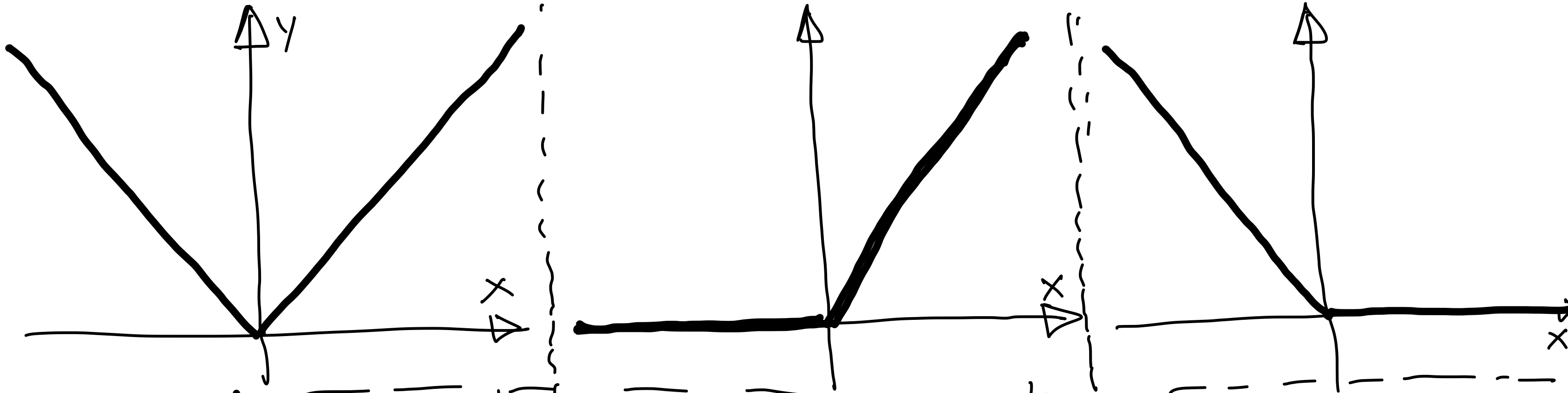
$\sum_{n=1}^{\infty} a_n$ given series

If $\sum_{n=1}^{\infty} |a_n|$ series converges, then

we say that $\sum_{n=1}^{\infty} a_n$ CONVERGES ABSOLUTELY

Thm If $\sum_{n=1}^{\infty} |a_n|$ CONERGES, then

$\sum_{n=1}^{\infty} a_n$ converges (Absolute convergence $\Rightarrow$ convergence)



abs(x) $|x|$ $|x|_+$ $|x|_-$

$|x| = |x|_+ + |x|_-$ $\quad$ $|x|_+ \geq 0$, $|x|_- \geq 0$

$x = |x|_+ - |x|_-$ $\quad$ $|x| \geq |x|_+$

$|x| \geq |x|_-$

Proof: Assume $\sum_{n=1}^{\infty} |a_n|$ converges.

$$0 \le |a_k|_+ \le |a_k| \Rightarrow \underbrace{\sum_{k=1}^{n} |a_k|_+}_{\forall n \in \mathbb{N}} \le \sum_{k=1}^{n} |a_k| \le \sum_{k=1}^{\infty} |a_k|$$

$\to$ nondecreasing bdd

MCT $\sum_{k=1}^{\infty} |a_k|_+$ converges

$$0 \le |a_k|_- \le |a_k| \Rightarrow \sum_{k=1}^{\infty} |a_k|_- \text{ converges.}$$

Algebra of Convergent Series implies that

$$\sum_{k=1}^{\infty} \big(\underbrace{|a_k|_+ - |a_k|_-}_{= a_k}\big) \text{ also converges}$$

Thus $\sum_{k=1}^{\infty} a_k$ converges

How to test ABSOLUTE CONVERGENCE?

Use The RATIO TEST!

(Is a series a geometrish SERIES?)

$\frac{|a_{n+1}|}{|a_n|}$ is this constant? (Is it constantish?)

no it is not

Does $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$ exist? $= L$

$L<1 \Rightarrow \sum |a_n|$ converges.

$L>1 \Rightarrow \sum |a_n|$ diverges.

$\sum_{n=1}^{\infty} a_n$ converges absolutely provided that $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} < 1$

Consider the series

$$\sum_{n=0}^{\infty} \frac{1}{n!} x^n \qquad x = 1 \quad \sum_{n=0}^{\infty} \frac{1}{n!} = e$$

$x \in \mathbb{R}$, variable

Do ratio test!

$$\lim_{n\to\infty} \frac{\frac{1}{(n+1)!}|x|^{n+1}}{\frac{1}{n!}|x|^n} = |x| \lim_{n\to+\infty} \frac{1}{n+1} = 0 < 1$$

$\forall x \in \mathbb{R}$

$$\sum_{n=0}^{\infty} \frac{1}{n!} x^n = e^x$$

$\boxed{\text{Converges absolutely.}}$

Series representation

$$\left(1+\frac{1}{n}\right)^n - \sum_{k=0}^{n} \frac{1}{k!} \to 0$$

x $\qquad$ x^k

$$\left(\left(1+\frac{x}{n}\right)^{\frac{n}{x}}\right)^x \to e^x$$

Use $\boxed{e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n}$

to understant (prove)

$$e^{ix} = \cos x + i(\sin x)$$

$i = \sqrt{-1}$ imaginary unit

complex number $(a+ib)+(c+id)$

$*$

$/$

i^{i} ? ?

$e^{ix} = \cos x + i(\sin x)$