MATH 302 Assignment 2 May 21, 2009

Name ____________________

Problem 1. Do problem 2.7.3.

Problem 2. Let $n \in \{2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$. Prove that among any n consecutive integers there is always at least one integer relatively prime to the other $n-1$ integers.

Proving the above statement for $n = 2$ should be very simple. Proofs get harder for a larger number n. Present only one proof. If you decide to do a larger n your successful effort will be rewarded with extra credit according to the formula

$$\text{The credit for this problem} = \frac{n}{4}.$$

Problem 3. Let $a,b,c,n \in \mathbb{Z}$ and $n > 1$. Set $d = \gcd(c,n)$ and $n = dm$.
Prove: If $ac \equiv bc \pmod{n}$, then $a \equiv b \pmod{m}$.

Problem 4. For which integers c with $0 \leq c < 1001$ does the congruence $154x \equiv c \pmod{1001}$ have solutions? Explain your reasoning. For each case when there are solutions find how many solutions are not congruent modulo 1001.

In the next two problems the following two identities will be useful.
Let b and k be integers and $k \geq 1$, then

$$b^{k+1} - 1 = (b-1)(b^k + b^{k-1} + \cdots + b + 1) = (b-1)\sum_{j=0}^{k} b^j \ .$$

Let b and k be integers and $k \geq 1$, then

$$b^{2k+1} + 1 = (b+1)(b^{2k} - b^{2k-1} + \cdots - b + 1) = (b+1)\sum_{j=0}^{2k} (-1)^j b^j \ .$$

Problem 5. Let a and n be integers such that $n \geq 1$ and $a \geq 2$.
Prove: If $a^n + 1$ is prime, then there exists an integer m such that $n = 2^m$.

Problem 6. Let a and n be integers such that $n > 1$ and $a \geq 2$.
Prove: If $a^n - 1$ is prime, then $a = 2$ and n is prime.