4) If $a$ and $b$ are odd perfect squares, then $a+b$ is not a perfect square.

$$a = (2k+1)^2 = 4k^2 + 4k + 1$$
$$b = (2\ell+1)^2 = 4\ell^2 + 4\ell + 1$$

$a \in \mathbb{O}$, $b \in \mathbb{O}$ $\Rightarrow a+b \in \mathbb{E}$

$$a+b = 4k^2 + 4k + 1 + 4\ell^2 + 4\ell + 1$$
$$= 4(k^2+\ell^2) + 4(k+\ell) + 2 \in \mathbb{E}$$

If $c^2 \in \mathbb{E} \Rightarrow c \in \mathbb{E}$

Proof:

Prove the contrapositive.

If $c \in \mathbb{O}$ $\Rightarrow c^2 \in \mathbb{O}$

$(2n+1)^2 = 4n^2+4n+1$

$n \in \mathbb{Z}$

$= 2(2n^2+2n)+1$

$\Rightarrow c^2 \in \mathbb{O}$

Does $a+b = (2m)^2 = 4m^2$; for $m \in \mathbb{Z}$

No! $4 \nmid (a+b)$

$\Rightarrow a+b$ is not a perfect square.