The first step towards a construction of the Cesaro fractal

May 6, 2020

We can think of $\overline{AB}$ as being of length 1
We need to calculate x and y as functions of α. $\alpha \in [0, \pi/2]$

$$x + y = 1/2$$

$$\sin\alpha = \frac{y}{x} = \frac{\text{opposite leg}}{\text{hypothenus}}$$

$$y = x\sin\alpha$$

$$x + x\sin\alpha = 1/2$$

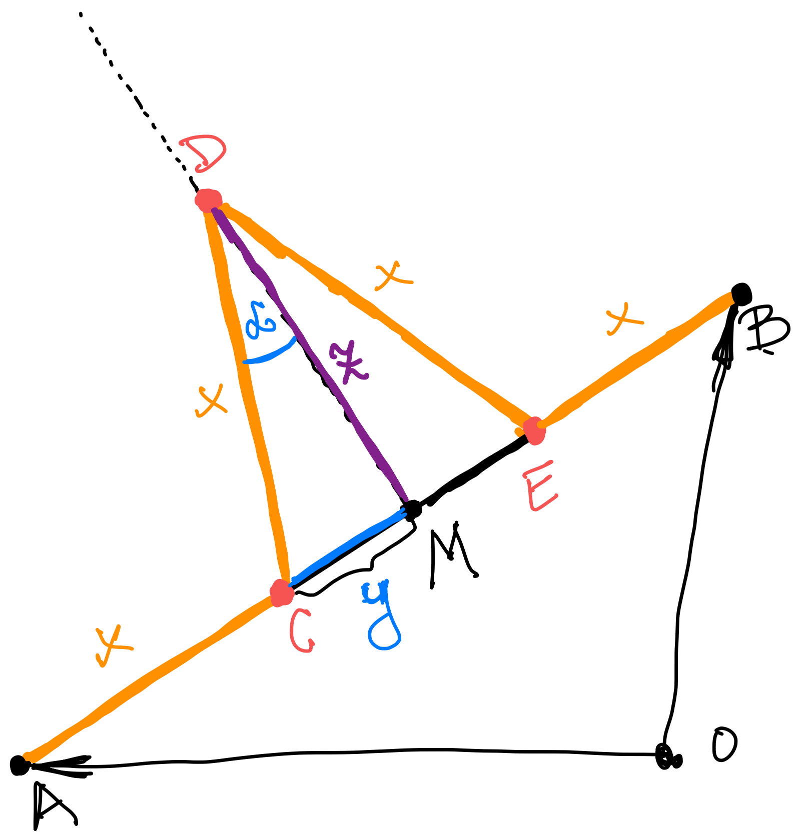
$$\boxed{x = \frac{1/2}{1+\sin\alpha}}$$

$$y = \frac{1}{2} - \frac{1}{2}\frac{1}{1+\sin\alpha} = \frac{1}{2}\frac{1+\sin\alpha - 1}{1+\sin\alpha}$$

$$\boxed{y = \frac{1}{2}\frac{\sin\alpha}{1+\sin\alpha}}$$

We also need $z = \sqrt{x^2 - y^2}$

$$z = \frac{1/2}{(1+\sin\alpha)}\sqrt{1-(\sin\alpha)^2} = \boxed{\frac{1}{2}\frac{\cos\alpha}{1+\sin\alpha}}$$



Now we can define the points pC, pD, pE based on pA, pB

$$pC = pA + x(pB - pA)$$

$$pD = \frac{1}{2}pA + \frac{1}{2}pB + z\{\{0,-1\},\{1,0\}\}.(pB - pA)$$

$$pE = pB - x(pB - pA)$$

The above formulas are all dependent on $\alpha \in [0, \pi/2]$. Therefore, our formula for Ces1[{pA_, pB_}, α_] should depend on α. Then you can do your testing in Manipulate[ ] where you can vary α in the interval $[0, \pi/2]$. It should be easy to conclude where the points pC, pD and pE should be when $\alpha = 0$ and $\alpha = \pi/2$.