Spring 2015 Math 309 Topics for the Final Exam

Logic. Know:

- Truth table of the negation operator, conjunction, disjunction, exclusive disjunction, implication and biconditional.
- How to form the negation of an implication and contrapositive, converse, and inverse of an implication
- All different ways of saying p implies q
- How to prove tautologies, contradictions and logical equivalences using truth tables
- Logical equivalences, in particular distributive laws, De Morgan's laws and equivalences involving implications
- The meaning of the universal and the existential quantifier, and their negations
- How to work with nested quantifiers (how to state negations, how to recognize whether a statement is true or false and justify it, Exercises 26–33, 37, 38 and exercises on the web-site posted on April 4, 2017)
- The most important rules of inference: modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and the rules of inference for quantified statements
- Proofs from Section 1.5 related to odd/even integers, rational and irrational numbers (Example 14, Example 18, Example 19, Example 21, Example 24, and the corresponding Exercises 20–30)
- A direct proof that $\sqrt{2}$ is irrational posted on April 7, 2017.
- How to translate English sentences into logical propositions

Sets and Functions. Know

- The concept of a set, equality of sets, the concept of a subset, the empty set, cardinality of a finite set, the power set, Cartesian product
- Different set notations, set builder notation, use of ellipses, Venn diagrams,
- Set operations: intersection, union, set difference, complement, symmetric difference, and the corresponding set identities
- Proving set identities using a membership table
- The formal definition of a function (web-site) and the concepts of domain, codomain and range
- Definitions of a surjection, an injection and a bijection; how to recognize and prove whether a given function has these properties (Exercises 12, 13, 14, 17, 18)
- The concept of composition of functions and the inverse function and connections to the previous item
- Properties of the floor and the ceiling and how to use them to solve related exercises (Examples 24, 25, Exercises 48, 49, 65, 66)

Axioms and Propositions for $\mathbb{Z}$. Know (The numbers in this section relate to the document "Basic properties of the Integers" posted on the class website)

- Section 2, Propositions 2.1, 2.2, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9
- Section 3, Propositions 3.1, 3.2, 3.6, Corollaries: 3.7, 3.8, Definitions 3.9 and 3.10, Exercises 3.11 and 3.12
- Section 4, Proposition 4.3

- Section 5, Theorem 5.1

Sequences, Induction and Recursion. Know

- Some common sequences, the basic properties of the summation notation
- The formulas for the sums of an arithmetic progression and a geometric progression with proofs
- The definition of a countable set and how to prove that the set $\mathbb{Z}$ is countable.
- The concept of cardinality for sets; how to prove that the sets $\mathcal{P}(S)$ and $\{0,1\}^S$ have the same cardinality (S is a nonempty set here); and that $\mathcal{P}(\mathbb{Z}^+)$ is not countable.
- The formal statement of the Principle of Mathematical Induction (and a proof from the notes "Basic properties of the Integers")
- How to do proofs involving both versions of the Mathematical Induction
- How recursive definitions work and proofs involving recursively defined functions

Counting. Know

- The basic counting principles and how to apply them to accurately count various sets
- How to apply the Pigeonhole principle in various situations
- Proof of Dirichelt's Approximation Theorem
- How to use permutations and combinations to count various sets
- Basic identities involving permutations and combinations and how to prove them using algebraic and combinatorial methods
- Generalized permutations and combinations, how to count combinations and permutations with repetition
- How to find closed form formula for a recursive sequence given by linear second-order homogeneous recursion (in book's language: How to solve linear second-order homogeneous recurrence relations) (Section 6.2: Theorem 1, Examples 3, 4, Theorem 2, Example 5, Exercises 3, 11)