Math 321: Week 1

Preliminaries

- Review of functions. Important concepts: domain and range. In this course all domains of functions are **intervals** of real numbers.
- Review of derivatives. Derivatives of elementary functions; the product rule and the chain rule; geometric interpretation of the derivative.

Product rule: $(u\,v)' = u\,v' + u'\,v$	Chain rule: $\dfrac{d}{dt}\Big[F\big(G(t)\big)\Big] = F'\big(G(t)\big)G'(t)$

Important special cases for the product rule:

(1) $u(t) = e^{2t},\ v(t) = y(t)$. Then $(e^t\,y)' = e^t\,y' + 2e^t\,y$.

(2) $u(t) = e^{t/2},\ v(t) = y(t)$. Then $(e^{t/2}\,y)' = e^{t/2}\,y' + \frac{1}{2}\,e^{t/2}\,y$.

(3) $u(t) = \sqrt{t},\ v(t) = y(t)$. Then $\big(\sqrt{t}\,y\big)' = \sqrt{t}\,y' + \dfrac{1}{2\sqrt{t}}\,y$.

Important special cases for the chain rule:

(1) $F(x) = \ln|x|,\ G(t) = y(t)$. Then $\dfrac{d}{dt}\Big[\ln\big|y(t)\big|\Big] = \dfrac{y'(t)}{y(t)} = \dfrac{y'}{y}$.

(2) $F(x) = x^2,\ G(t) = y(t)$. Then $\dfrac{d}{dt}\Big[\big(y(t)\big)^2\Big] = 2\,y(t)y'(t) = 2\,y\,y'$.

(3) $F(x) = \sqrt{x},\ G(t) = y(t)$. Then $\dfrac{d}{dt}\Big[\sqrt{y(t)}\Big] = \dfrac{1}{2}\dfrac{1}{\sqrt{y(t)}}\,y'(t) = \dfrac{1}{2\sqrt{y}}\,y'$.

- Review of integration: Table of integrals; integration by parts and integration by substitution.

What is a differential equation?

Basic concepts:

- direction field
- a solution (all solutions)
- initial value problem

Solving simple differential equations: (below m and k are constants)

$y' = 1$, $y' = t$, $y' = \cos t$, $y' = \arctan t$, $y' = 1/t$, $y' = t\,e^t$, $y' = t\,\sin t$,
$y' = y$, $y' = -y$, $y' = y/2$, $y' = m\,y$, $y' = m\,y + k$, $y' = 1/(2y)$, $y' = 2\sqrt{y}$,,

Linear Differential Equations (Section 2.1)

- I will demonstrate in class how to draw direction fields using the computer algebra system *Mathamatica*.
- Do the following exercises:
  2, 3, 4, 5, 6, 7, 9, 10, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 31, 32, 35, 36