Burgers' Equation

PDE $\quad \underbrace{u(x,t)}\, u_x(x,t) + u_t(x,t) = 0$

I.C. $\quad u(x,0) = f(x) \qquad x \in \mathbb{R}$

The vector field for the charact. eqs is

$$(x,t,z) \longmapsto \boxed{\langle z, 1, 0 \rangle}$$

characteristic equations:

(seek the $\boxed{\text{integral curves}}$ of the above vec. field)

$$\underbrace{\langle X(s), T(s), Z(s) \rangle}$$

this curve

tangent

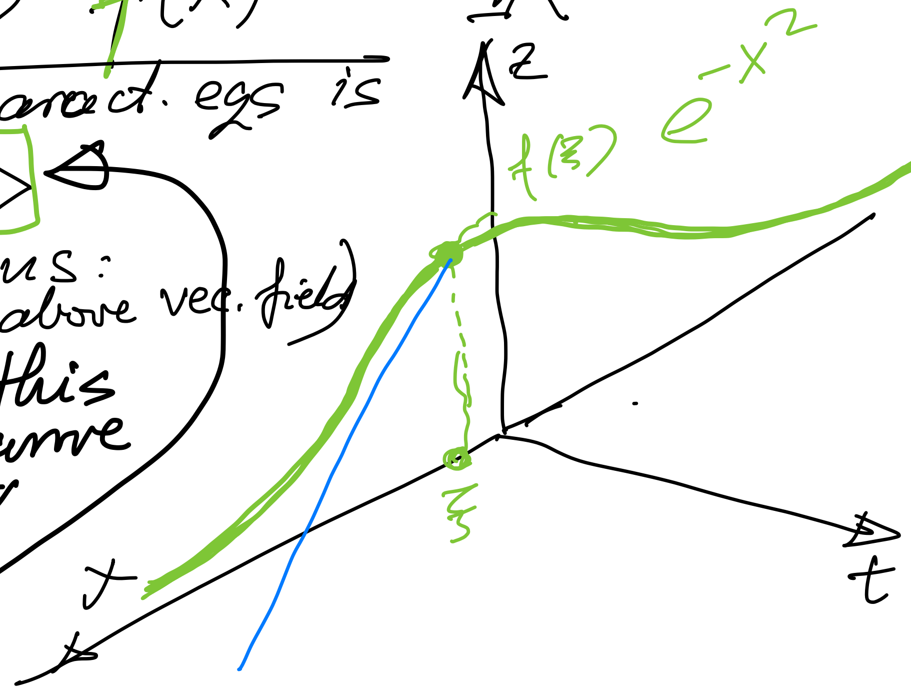
to

$$X'(s) = Z(s)$$

$$T'(s) = 1$$

$$Z'(s) = 0$$

$$X(0) = \xi, \quad T(0) = 0, \quad Z(0) = f(\xi)$$



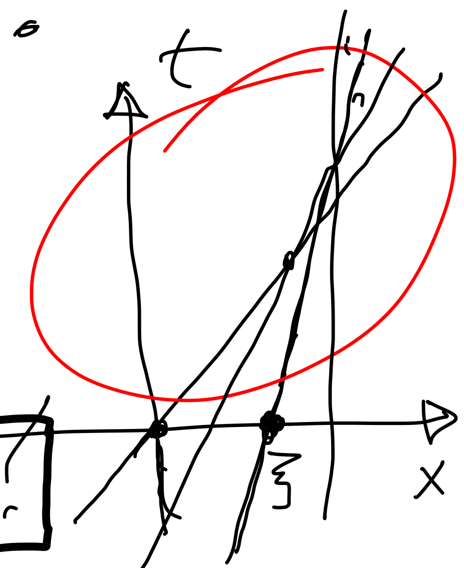
$$\left.\begin{array}{l} T(s) = s \\ z(s) = f(\xi) \\ x(s) = f(\xi)s + \xi \end{array}\right\} \text{So}$$

$\langle f(\xi)s+\xi,\ s,\ f(\xi)\rangle$, $s \in \mathbb{R}$ line in S which starts at $(\xi, 0, f(\xi))$

To understand the blue lines (the characteristics) we study their projections onto xt-plane:

$$\langle \underbrace{f(\xi)s+\xi}_{x},\ \underset{t}{\overset{\shortparallel}{s}} \rangle$$

$$x = f(\xi)t + \xi$$



$\boxed{\text{We move to the computer!}}$

Math 225
Math 331

$\mathbb{R}^2$ xy-space $(x, y) \longmapsto \langle -y, x \rangle$

$\langle X'(s), Y'(s) \rangle$

$X'(s) = -Y(s)$

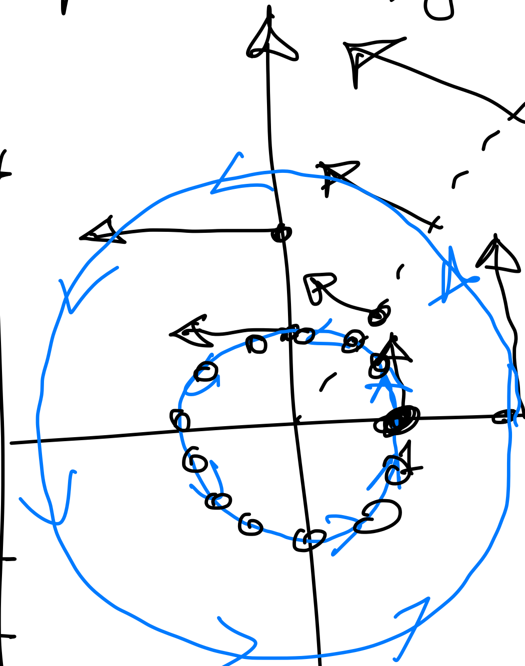
$Y'(s) = X(s)$

$Z'(s) = 1$

$X(0) = \xi, Y(0) = 0, Z(0) = 0$

$\langle \cos(s), \sin(s), s \rangle$

$X(0) = \cos\xi, \; Y(0) = \sin\xi, \; Z(0) = 0$



$X'(s) = -Y(s)$

$Y'(s) = X(s)$

$X(0) = 1$

$Y(0) = 0$

$X(s) = \cos s$

$Y(s) = \sin s$