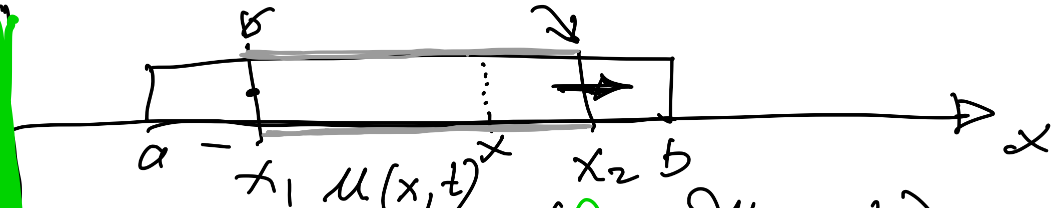
Diffusion & Heat Equation

Cons. of dye Law

$$\frac{d}{dt}\int_{x_1}^{x_2} u(\xi,t)\,d\xi = -\phi(x_2,t)+\phi(x_1,t)$$

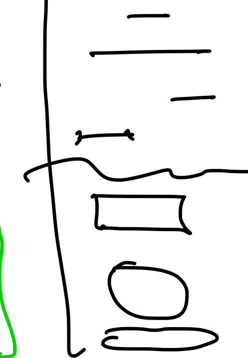


$$\frac{\partial u}{\partial t}(x,t) = \frac{\partial}{\partial x}\left(k(x)\,\frac{\partial u}{\partial x}(x,t)\right)$$

Fick's Law

$$\boxed{\phi(x,t) = -k(x)\,\frac{\partial u}{\partial x}(x,t)}$$

vector quantity



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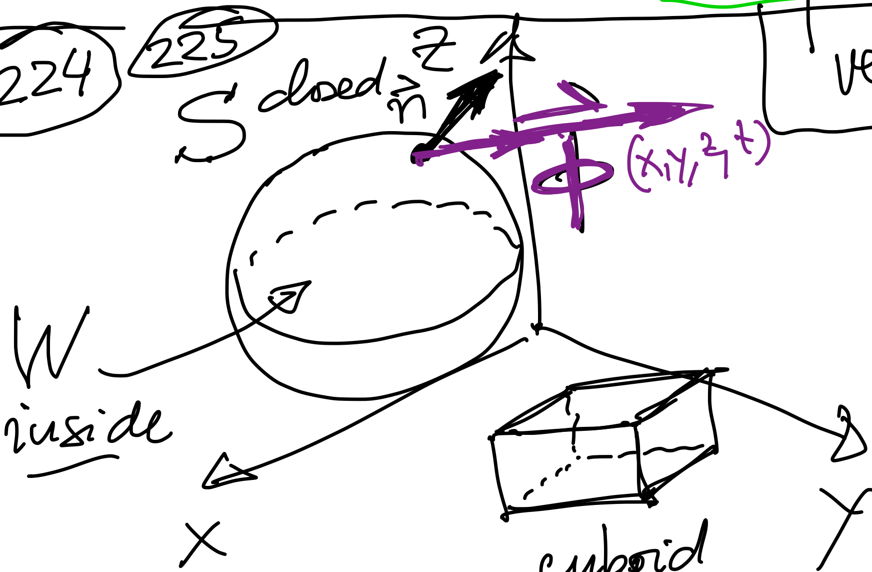


The unknown function is

$$u(\underbrace{x,y,z}_{\mathbf{x}},t) = u(\mathbf{x},t)$$

concentration of dye

$T(x)$ you know

$T'(x) > 0$

touch it — it's <u>cold</u>

$T'(x) < 0$

Fick's <u>Law</u> in 3-dim

$$\boxed{\vec{\Phi}(\mathbf{x},t) = -k\,\vec{\nabla} u(\mathbf{x},t)}$$

$\mathbf{x} = (x,y,z)$

k fixed or dep. on $\mathbf{x}$

$F(x,y,z)$ scalar function $\quad \vec{\nabla} F = \left\langle \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right\rangle$

$\vec{\nabla} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \quad$ "scaling of $\vec{\nabla}$ by scalar F"

Conservation of dye Law

$$\frac{d}{dt} \iiint_W u(x,y,z,t)\, dV = - \oiint_S \vec{\phi}(x,y,z,t) \cdot \vec{n}\, dS$$

dV: $dx\,dy\,dz$ element of volume

$$\oiint_S \vec{F}(x,y,z) \cdot \vec{n}\, dA = \iiint_W (\operatorname{div} \vec{F})(x,y,z)\, dV$$

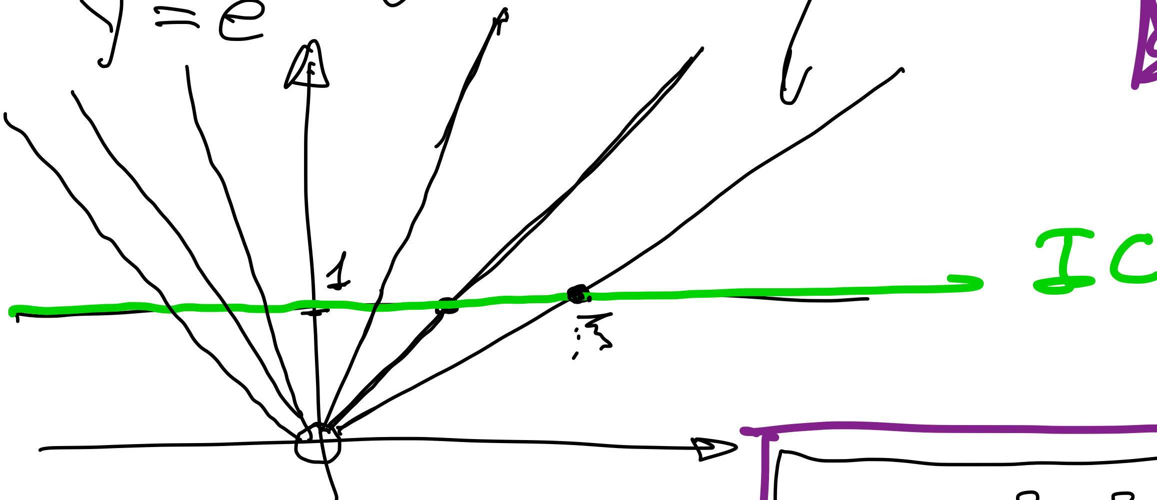
$\cancel{\frac{\partial F}{\partial x}}$ $\quad \vec{\nabla} \cdot \vec{F}$

$$\vec{F} = \langle F_1, F_2, F_3 \rangle$$

$$\frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$\left.\begin{aligned} x &= \xi e^{s} \\ y &= e^{s} \end{aligned}\right\} \quad x = \xi y$$

$$x u_x + y u_y = \underbrace{2u \ln u}_{2(x^2-y^2)e^{x^2-y^2}} \quad \checkmark$$



$y = 1$
$x = \xi$

$$\boxed{\begin{aligned} x &= \xi e^{s} \\ y &= e^{s} \\ z &= e^{e^{2s}(\xi^2 - 1)} \end{aligned}}$$

$$\boxed{u = e^{x^2 - y^2}}$$

It, solves it.

by chance