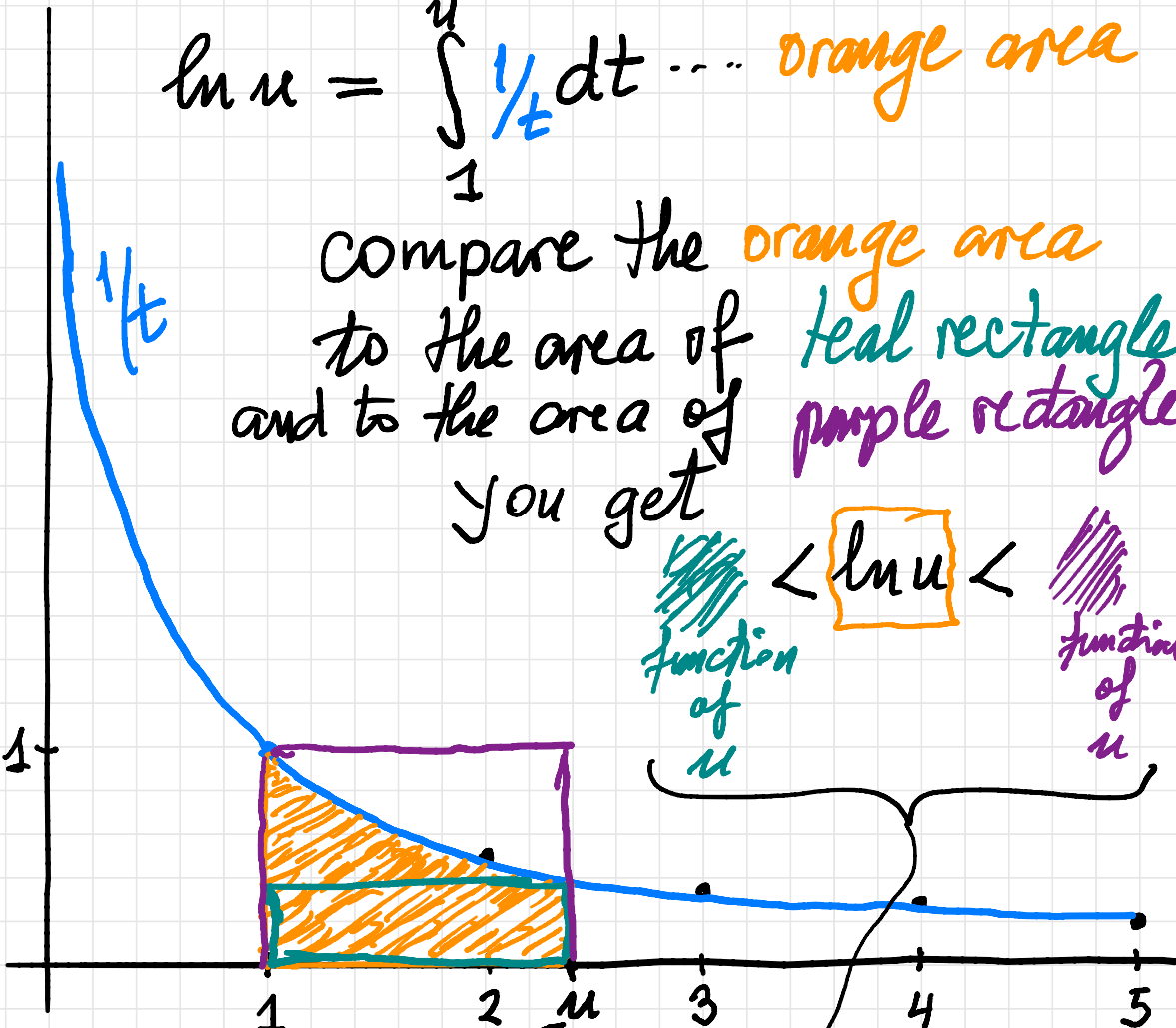


$$\ln u = \int_1^u \frac{1}{t} dt \dots \text{orange area}$$

compare the orange area
to the area of teal rectangle
and to the area of purple rectangle
you get

$$\text{function of } u < \boxed{\ln u} < \text{function of } u$$



$$u = 1 + \frac{1}{x} \text{ apply to}$$

$$\text{finally } \text{sum of } x < \ln \left(1 + \frac{1}{x} \right) < \text{sum of } x$$

$$\text{finally } \text{sum of } x < \ln \left(1 + \frac{1}{x} \right)^x < \text{sum of } x$$