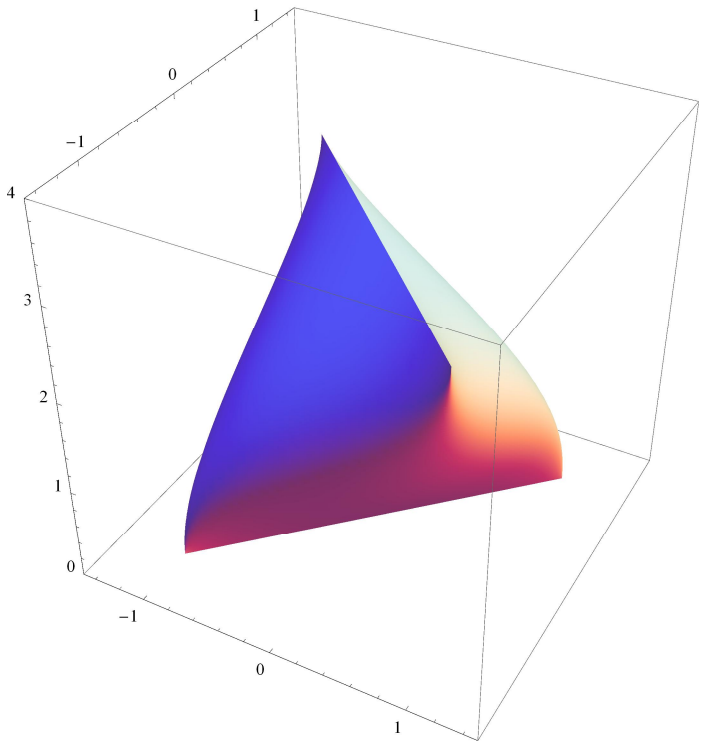
Some closed surfaces, their surface areas and volumes

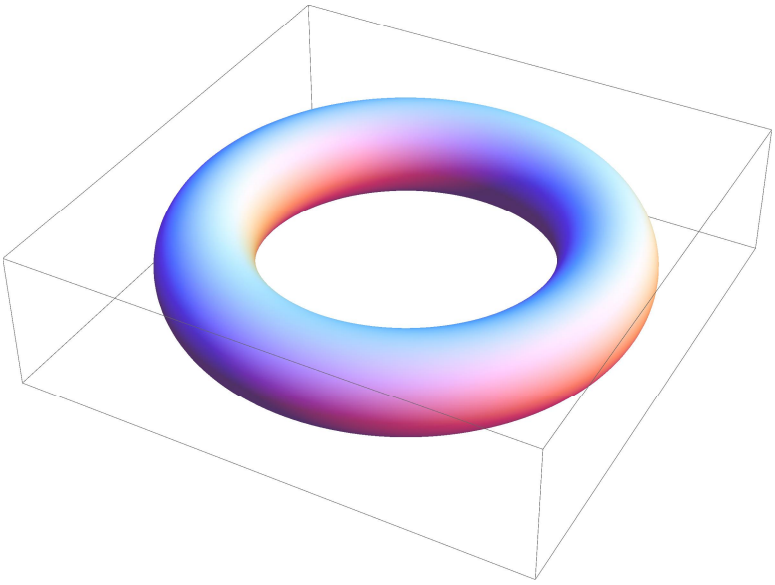
A fortune cookie

```
ParametricPlot3D[{Cos[t], Cos[t + s], s},
 {t, 0, 2 π}, {s, 0, π}, PlotPoints → {61, 91}, Mesh → None,
 PlotRange → {{-1.5`, 1.5`}, {-1.5`, 1.5`}, {0, 4}}, Axes → True, BoxRatios → {1, 1, 1}]
```

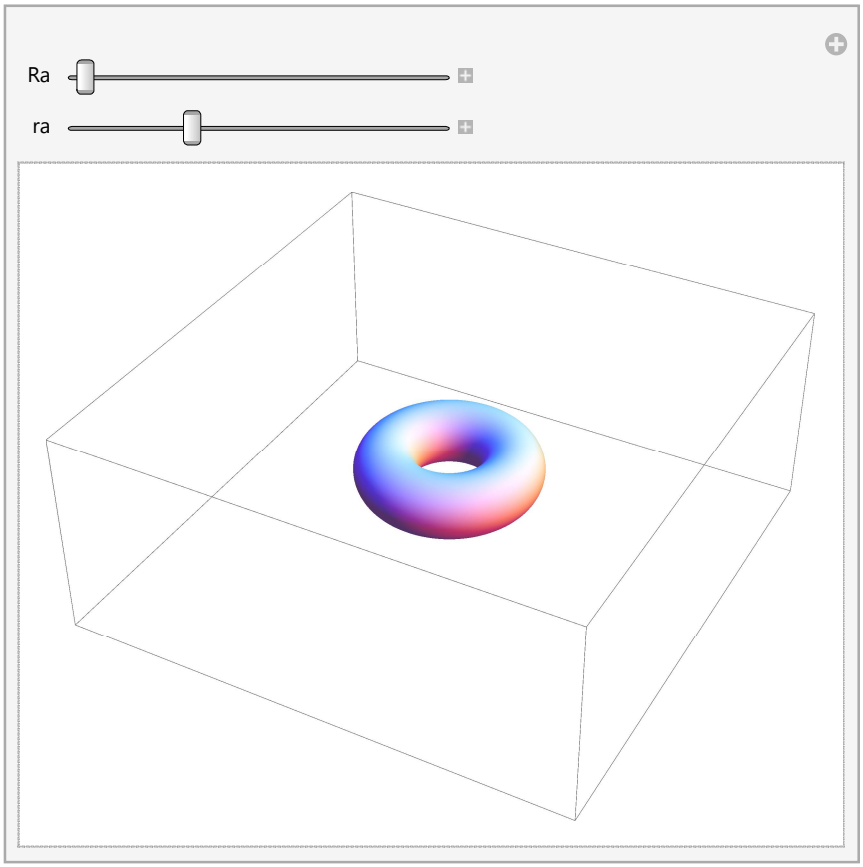


A Torus

```
Clear[Ra, ra]; Ra = 4; ra = 1;
ParametricPlot3D[{Cos[th] (Ra + ra Cos[ph]), Sin[th] (Ra + ra Cos[ph]), ra Sin[ph]},
 {th, 0, 2 π}, {ph, 0, 2 π}, PlotPoints → {161, 51}, Mesh → False, PlotRange →
  {{-Ra - ra - 0.5`, Ra + ra + 0.5`}, {-Ra - ra - 0.5`, Ra + ra + 0.5`}, {-ra - 0.5`, ra + 0.5`}},
 Axes → False, BoxRatios → Automatic, ImageSize → 400]
```

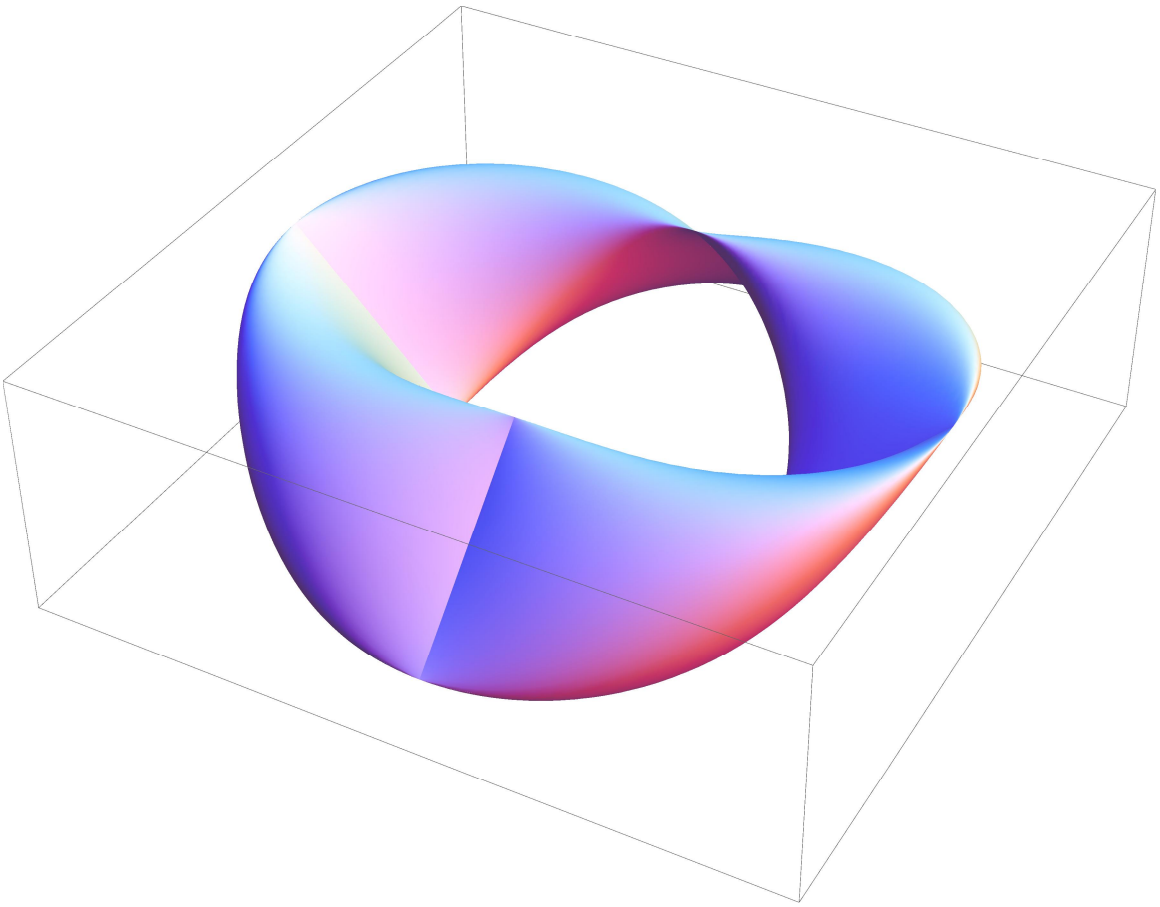


```
Clear[Ra, ra];
Manipulate[ParametricPlot3D[{Cos[th] (Ra + ra Cos[ph]), Sin[th] (Ra + ra Cos[ph]), ra Sin[ph]},
  {th, 0, 2 π}, {ph, 0, 2 π}, PlotPoints → {161, 51}, Mesh → False,
  PlotRange → {{-5 - 3 - 0.5`, 5 + 3 + 0.5`}, {-5 - 3 - 0.5`, 5 + 3 + 0.5`}, {-3 - 0.5`, 3 + 0.5`}},
  Axes → False, BoxRatios → Automatic, ImageSize → 400], {{Ra, 2}, 2, 5}, {{ra, 1}, .1, 3}]
```



The following surface is some kind of combination of a torus and a fortune cookie:

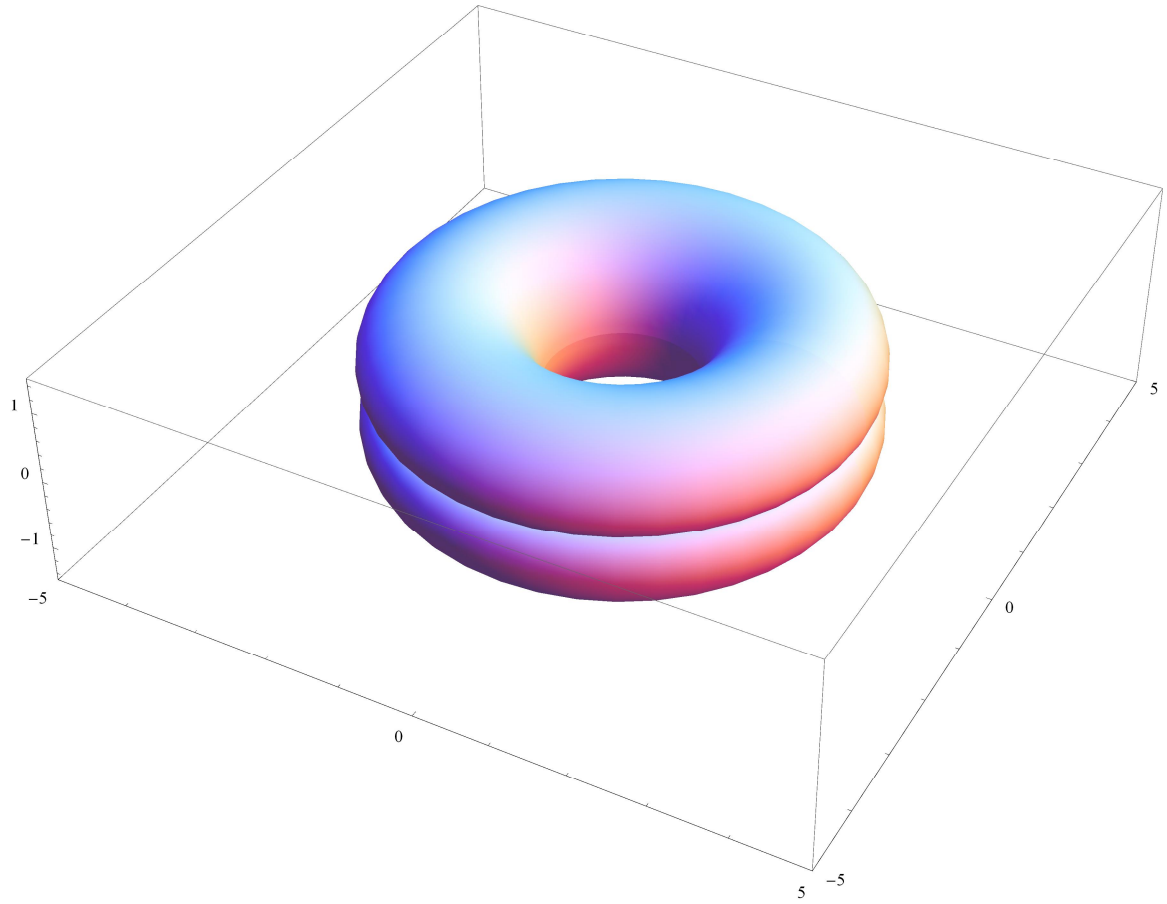
```
Clear[Ra, ra]; Ra = 3; ra = 1;
ParametricPlot3D[{Cos[th] (Ra + ra Cos[ph]), Sin[th] (Ra + ra Cos[ph]), ra Cos[ph + 2 th]},
 {th, 0, 2 π}, {ph, 0, 2 π}, PlotPoints → {161, 51}, Mesh → False, PlotRange →
  {{-Ra - ra - 0.5`, Ra + ra + 0.5`}, {-Ra - ra - 0.5`, Ra + ra + 0.5`}, {-ra - 0.5`, ra + 0.5`}},
 Axes → False, BoxRatios → Automatic, ImageSize → 600]
```



Several cardioid tori

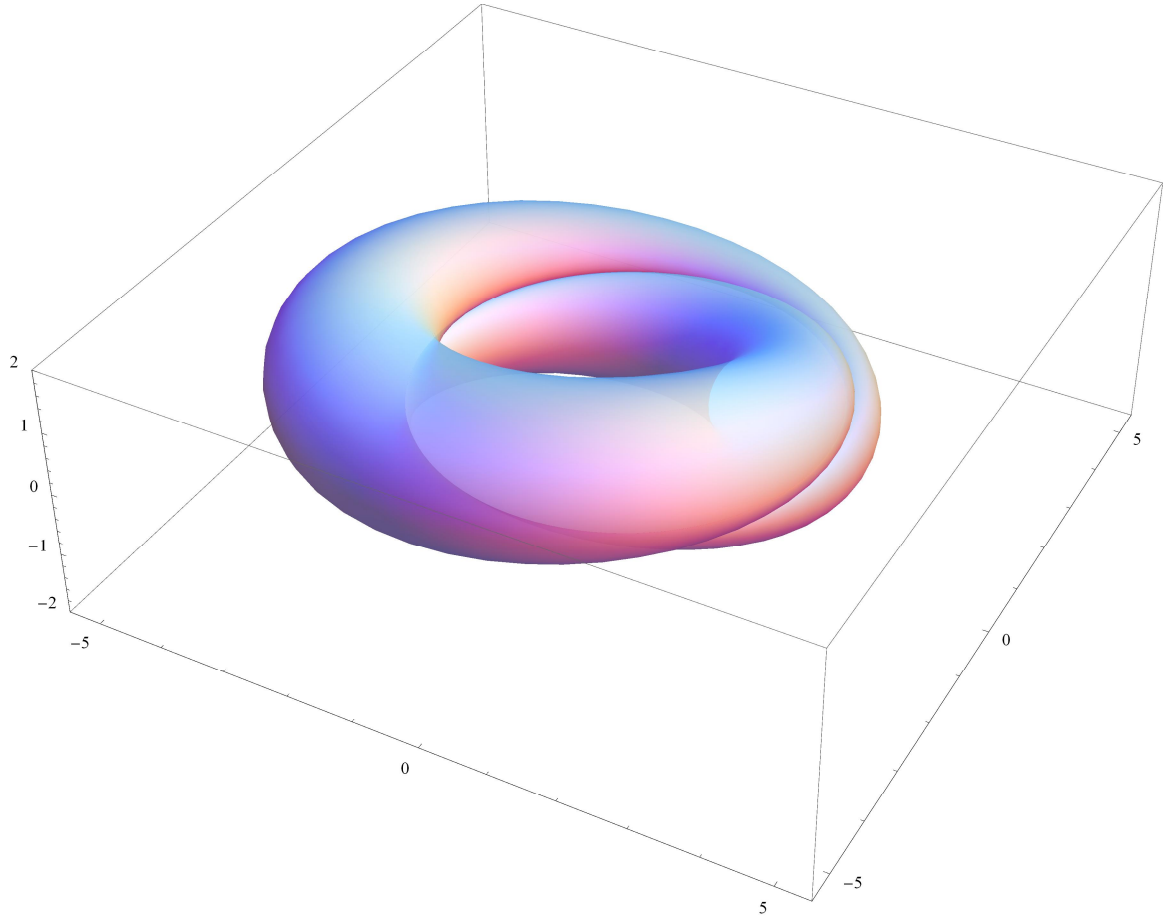
A simple cardioid torus

```
ParametricPlot3D[(3 - (1 + Cos[t]) Cos[t]) {Cos[θ], Sin[θ], 0} + (1 + Cos[t]) Sin[t] {0, 0, 1},
 {t, 0, 2 Pi}, {θ, 0, 2 Pi}, PlotPoints → {101, 41}, Mesh → False,
 PlotRange → {{-5, 5}, {-5, 5}, {-1.5, 1.5}}, Axes → True, ImageSize → 600]
```



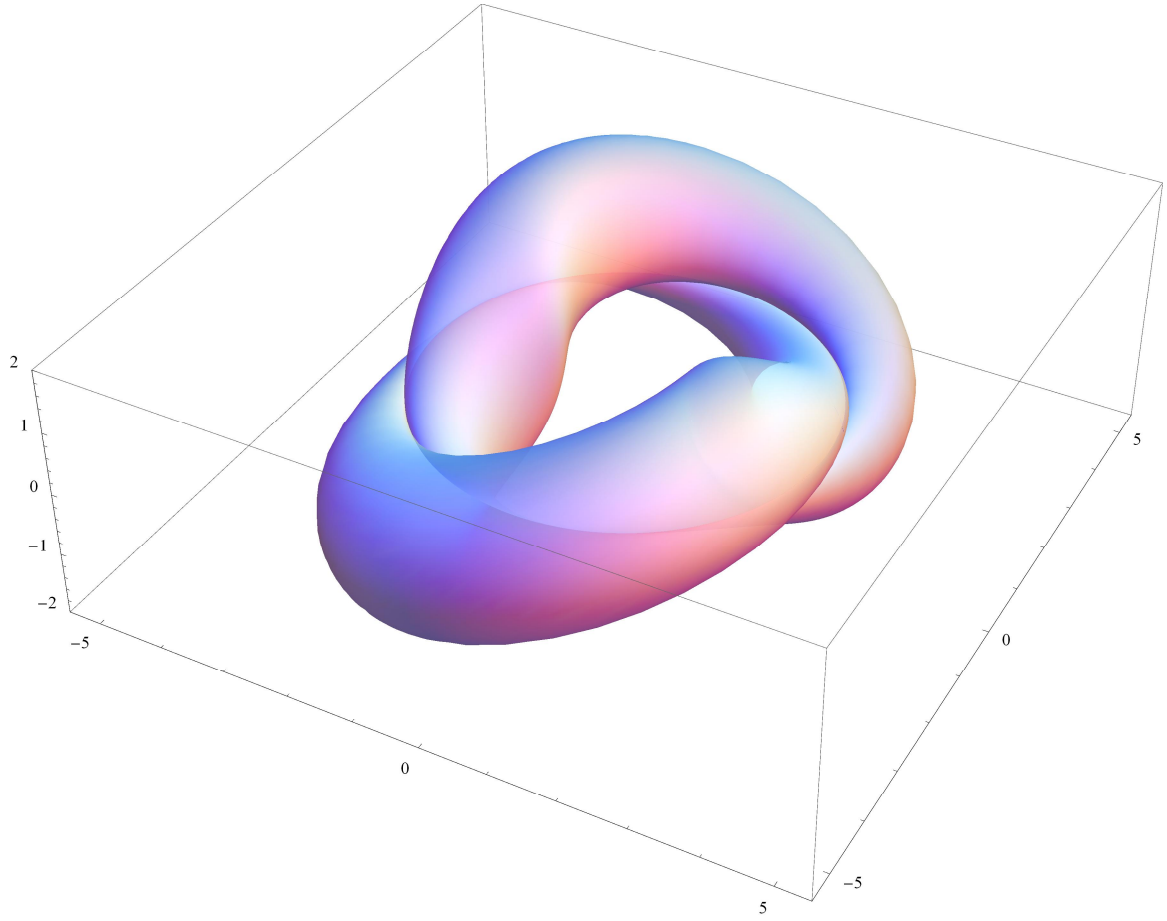
A simple cardioid torus with one rotation

```
nn = 1; ParametricPlot3D[3 {Cos[θ], Sin[θ], 0} +
   (- (1 + Cos[t]) Cos[t]) (Cos[nn θ] {Cos[θ], Sin[θ], 0} + Sin[nn θ] {0, 0, 1}) +
   (1 + Cos[t]) Sin[t] (- Sin[nn θ] {Cos[θ], Sin[θ], 0} + Cos[nn θ] {0, 0, 1}), {t, 0, 2 Pi},
 {θ, 0, 2 Pi}, PlotStyle → {Opacity[.8]}, PlotPoints → {50, 50}, Mesh → False,
 PlotRange → {{-5.5, 5.5}, {-5.5, 5.5}, {-2, 2}}, Boxed → True, Axes → True, ImageSize → 600]
```



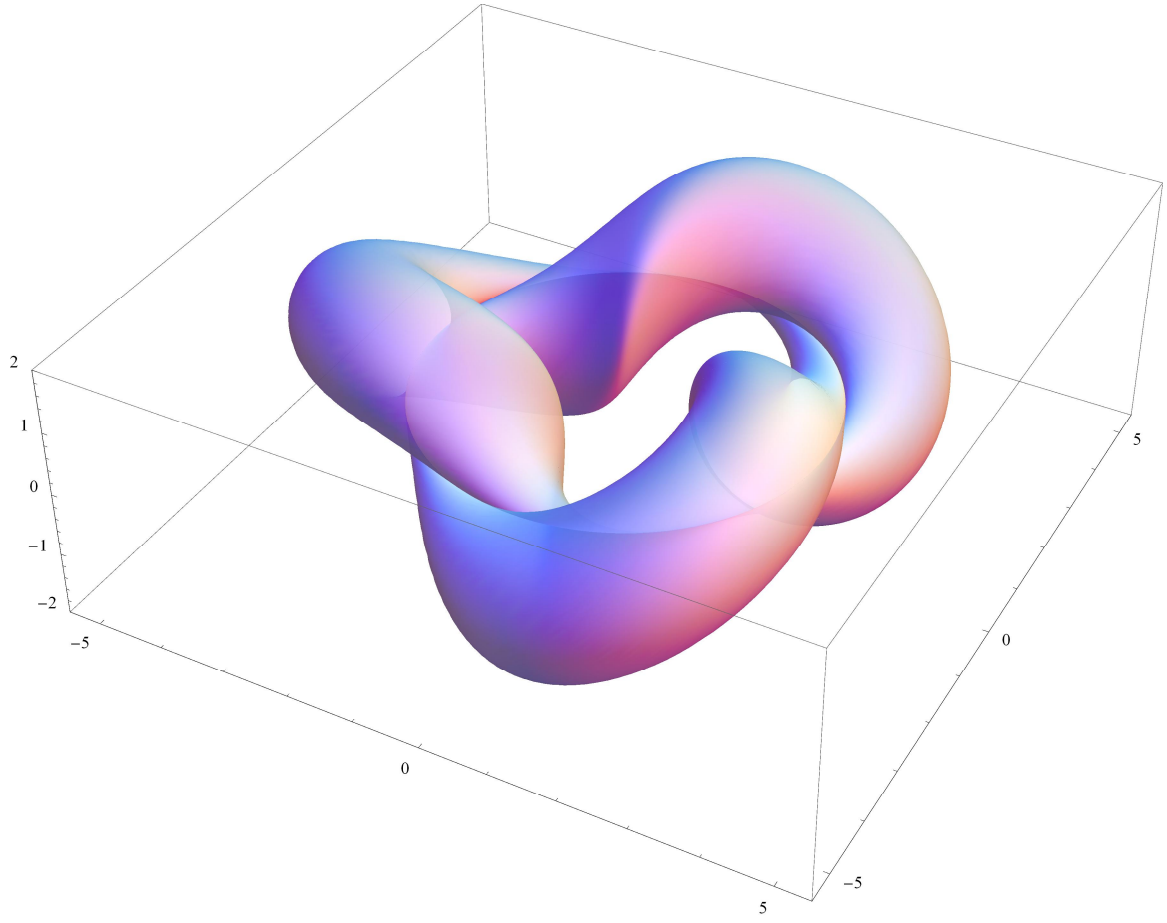
■ A simple cardioid torus with two rotation

```
nn = 2; ParametricPlot3D[3 {Cos[θ], Sin[θ], 0} +
   (- (1 + Cos[t]) Cos[t]) (Cos[nn θ] {Cos[θ], Sin[θ], 0} + Sin[nn θ] {0, 0, 1}) +
   (1 + Cos[t]) Sin[t] (- Sin[nn θ] {Cos[θ], Sin[θ], 0} + Cos[nn θ] {0, 0, 1}), {t, 0, 2 Pi},
 {θ, 0, 2 Pi}, PlotStyle → {Opacity[.8]}, PlotPoints → {50, 50}, Mesh → False,
 PlotRange → {{-5.5, 5.5}, {-5.5, 5.5}, {-2, 2}}, Boxed → True, Axes → True, ImageSize → 600]
```



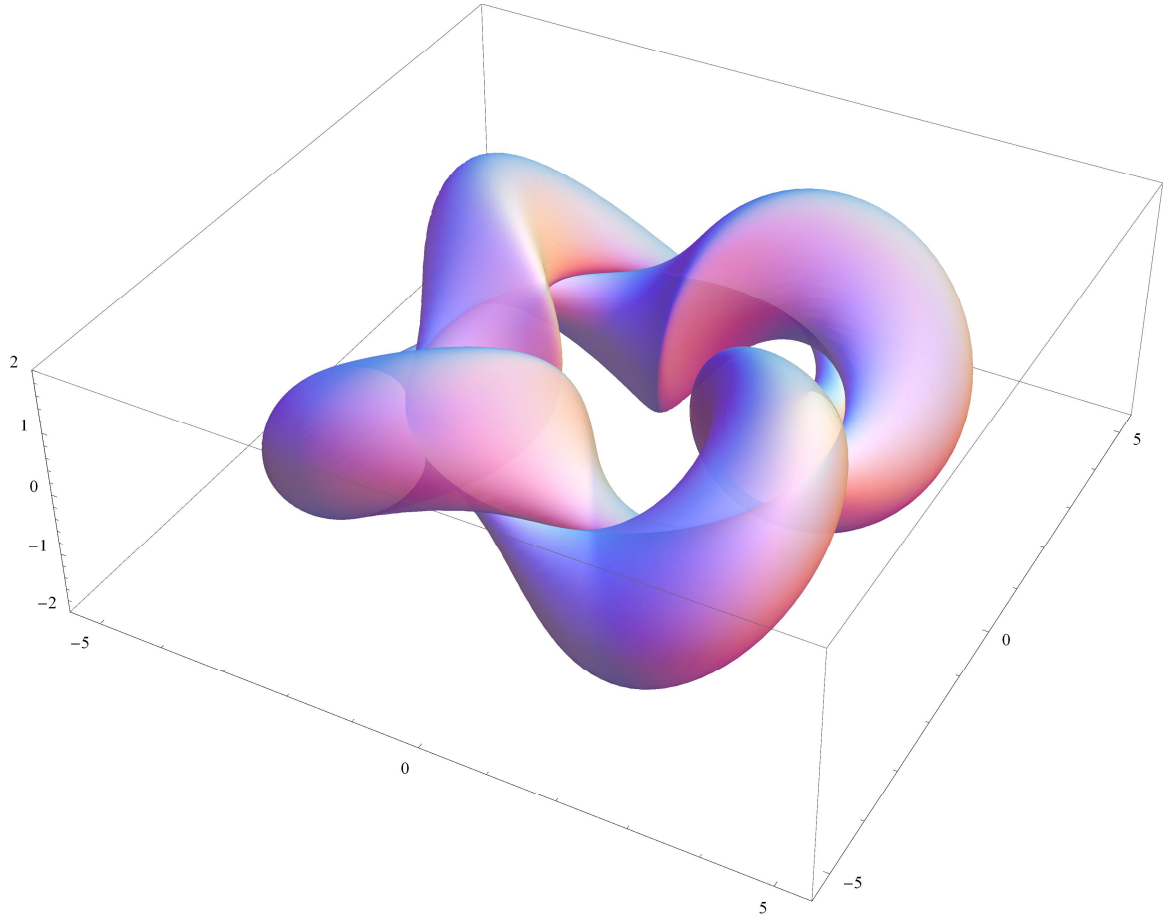
A simple cardioid torus with three rotations

```
nn = 3; ParametricPlot3D[3 {Cos[θ], Sin[θ], 0} +
   (- (1 + Cos[t]) Cos[t]) (Cos[nn θ] {Cos[θ], Sin[θ], 0} + Sin[nn θ] {0, 0, 1}) +
   (1 + Cos[t]) Sin[t] (- Sin[nn θ] {Cos[θ], Sin[θ], 0} + Cos[nn θ] {0, 0, 1}), {t, 0, 2 Pi},
 {θ, 0, 2 Pi}, PlotStyle → {Opacity[.8]}, PlotPoints → {50, 50}, Mesh → False,
 PlotRange → {{-5.5, 5.5}, {-5.5, 5.5}, {-2, 2}}, Boxed → True, Axes → True, ImageSize → 600]
```



■ **A simple cardioid torus with four rotations**

```
nn = 4; ParametricPlot3D[3 {Cos[θ], Sin[θ], 0} +
   (- (1 + Cos[t]) Cos[t]) (Cos[nn θ] {Cos[θ], Sin[θ], 0} + Sin[nn θ] {0, 0, 1}) +
   (1 + Cos[t]) Sin[t] (- Sin[nn θ] {Cos[θ], Sin[θ], 0} + Cos[nn θ] {0, 0, 1}), {t, 0, 2 Pi},
 {θ, 0, 2 Pi}, PlotStyle → {Opacity[.8]}, PlotPoints → {50, 50}, Mesh → False,
 PlotRange → {{-5.5, 5.5}, {-5.5, 5.5}, {-2, 2}}, Boxed → True, Axes → True, ImageSize → 600]
```



Your tasks

For each of the surfaces above determine the exact, or if that is not possible the approximate, value for

■ **The surface area**

■ **The volume**