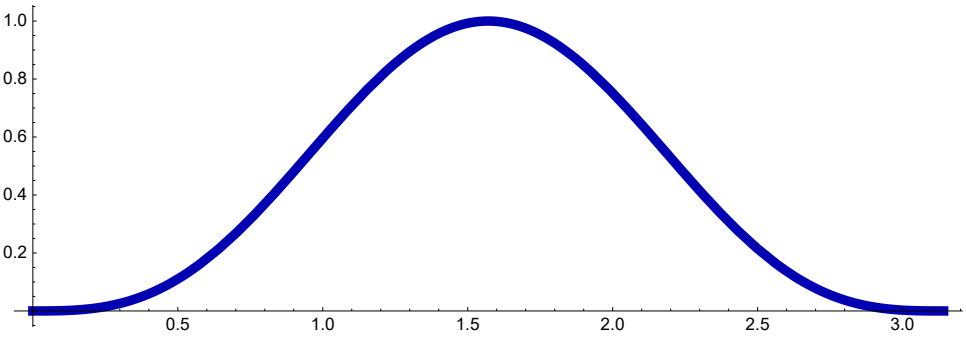
In[20]:=
```
Plot[Sin[x]^3, {x, 0, Pi}, PlotStyle → {RGBColor[0, 0, 0.7], Thickness[0.01]},
 AspectRatio → Automatic, ImageSize → 500]
```

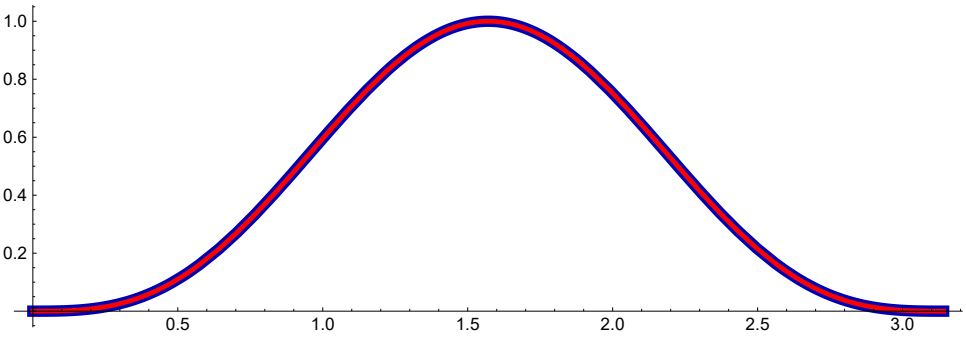
Out[20]=



From Wikipedia Power-reduction formula is

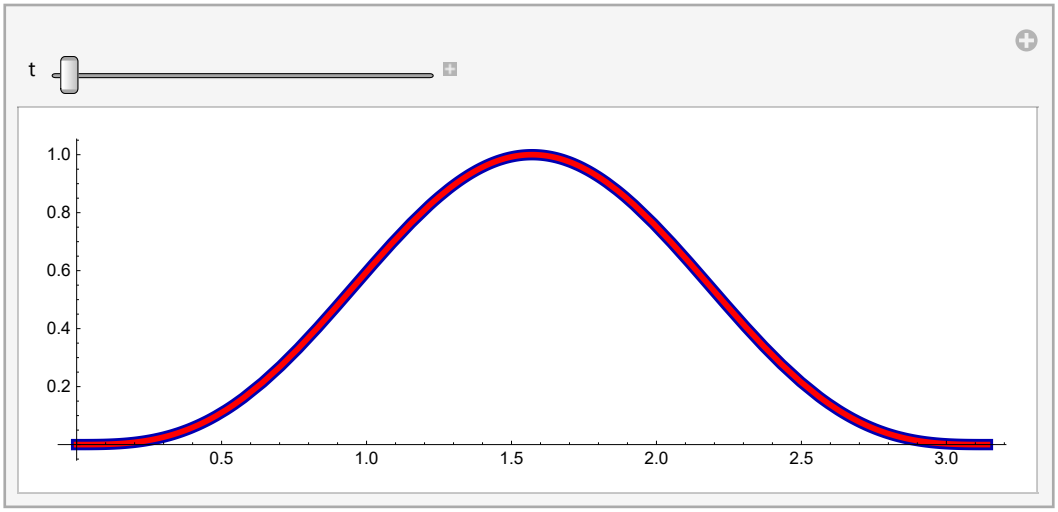
In[23]:=
```
Plot[{Sin[x]^3, 3/4 Sin[x] - 1/4 Sin[3 x]}, {x, 0, Pi}, PlotStyle →
  {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.005]}},
 AspectRatio → Automatic, ImageSize → 500]
```

Out[23]=



In[24]:=
```
Manipulate[
 Plot[{Sin[x]^3, 3/4 Exp[-t] Sin[x] - 1/4 Exp[-(3^2) t] Sin[3 x]}, {x, 0, Pi}, PlotStyle →
   {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.006]}},
  AspectRatio → Automatic, ImageSize → 500], {t, 0, 5}]
```

Out[24]=



In[3]:=
$$\frac{\text{Integrate}\left[\text{Sin}[x]^3\,\text{Sin}[x],\ \{x,\ 0,\ \text{Pi}\}\right]}{\frac{\text{Pi}}{2}}$$

Out[3]=
$$\frac{3}{4}$$

In[4]:=
$$\frac{\text{Integrate}\left[\text{Sin}[x]^3\,\text{Sin}[2\,x],\ \{x,\ 0,\ \text{Pi}\}\right]}{\frac{\text{Pi}}{2}}$$

Out[4]=
$$0$$

In[5]:=
$$\frac{\text{Integrate}\left[\text{Sin}[x]^3\,\text{Sin}[3\,x],\ \{x,\ 0,\ \text{Pi}\}\right]}{\frac{\text{Pi}}{2}}$$

Out[5]=
$$-\frac{1}{4}$$

In[6]:=
$$\text{Table}\left[\frac{\text{Integrate}\left[\text{Sin}[x]^3\,\text{Sin}[k\,x],\ \{x,\ 0,\ \text{Pi}\}\right]}{\frac{\text{Pi}}{2}},\ \{k,\ 1,\ 20\}\right]$$

Out[6]=
$$\left\{\frac{3}{4},\ 0,\ -\frac{1}{4},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\right\}$$

In[7]:=
```
Table[Integrate[Sin[x]^4 Sin[k x], {x, 0, Pi}] / (Pi/2), {k, 1, 20}]
```

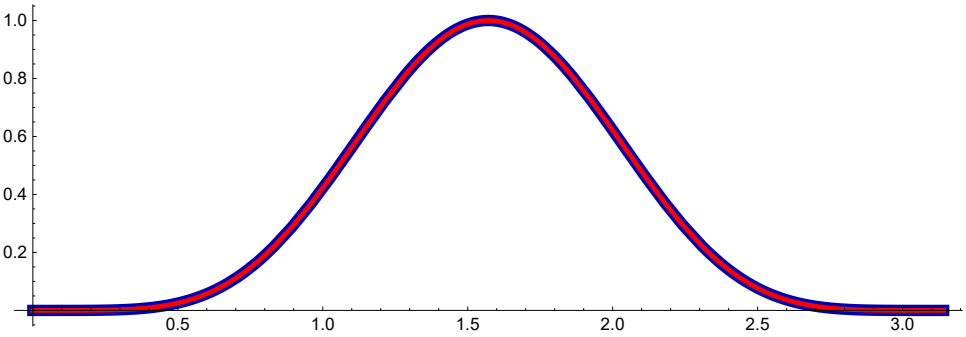
Out[7]=
$$\left\{\frac{32}{15\pi}, 0, -\frac{32}{35\pi}, 0, \frac{32}{315\pi}, 0, \frac{32}{3465\pi}, 0, \frac{32}{15015\pi}, 0, \frac{32}{45045\pi}, 0, \frac{32}{109395\pi}, 0, \frac{32}{230945\pi}, 0, \frac{32}{440895\pi}, 0, \frac{32}{780045\pi}, 0\right\}$$

In[8]:=
```
Table[Integrate[Sin[x]^5 Sin[k x], {x, 0, Pi}] / (Pi/2), {k, 1, 20}]
```

Out[8]=
$$\left\{\frac{5}{8}, 0, -\frac{5}{16}, 0, \frac{1}{16}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

In[26]:=
```
Plot[{Sin[x]^5, 5/8 Sin[x] - 5/16 Sin[3 x] + 1/16 Sin[5 x]}, {x, 0, Pi}, PlotStyle →
  {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.005]}},
 AspectRatio → Automatic, ImageSize → 500]
```

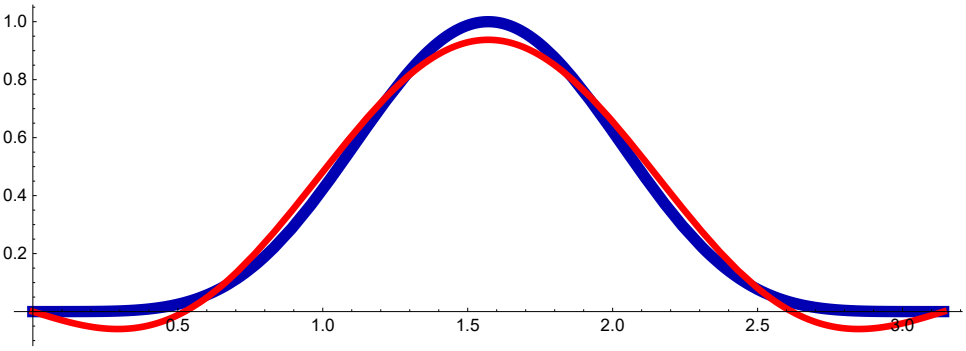
Out[26]=



Just to illustrate that even with two summands we get a good approximation for the blue function

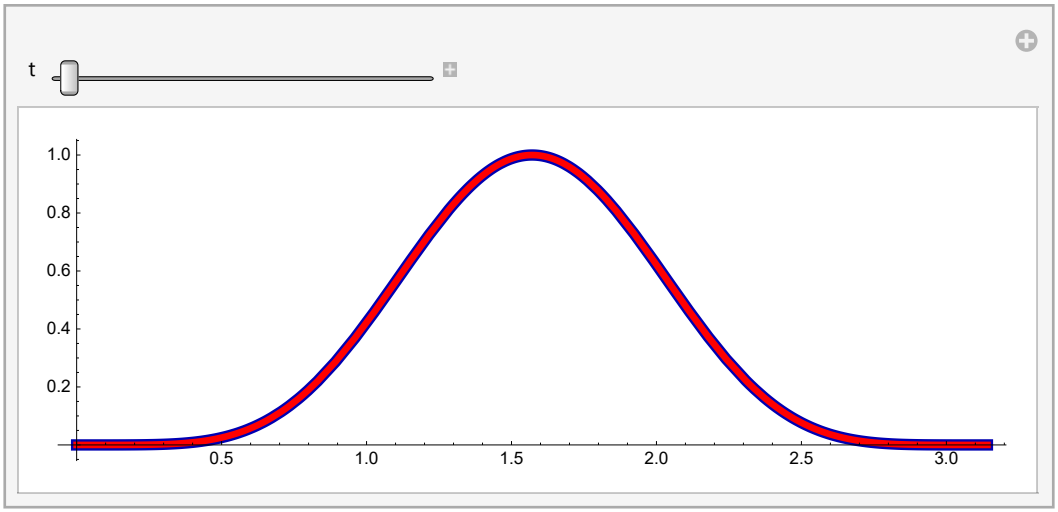
In[28]:=
```
Plot[{Sin[x]^5, 5/8 Sin[x] - 5/16 Sin[3 x] + 0 × 1/16 Sin[5 x]}, {x, 0, Pi}, PlotStyle →
  {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.007]}},
 AspectRatio → Automatic, ImageSize → 500]
```

Out[28]=



In[29]:=
```
Manipulate[
 Plot[{Sin[x]^5, 5/8 Exp[-t] Sin[x] - 5/16 Exp[-(3^2) t] Sin[3 x] + 1/16 Exp[-(5^2) t] Sin[5 x]},
  {x, 0, Pi}, PlotStyle →
   {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.007]}},
  AspectRatio → Automatic, ImageSize → 500], {t, 0, 5}]
```

Out[29]=



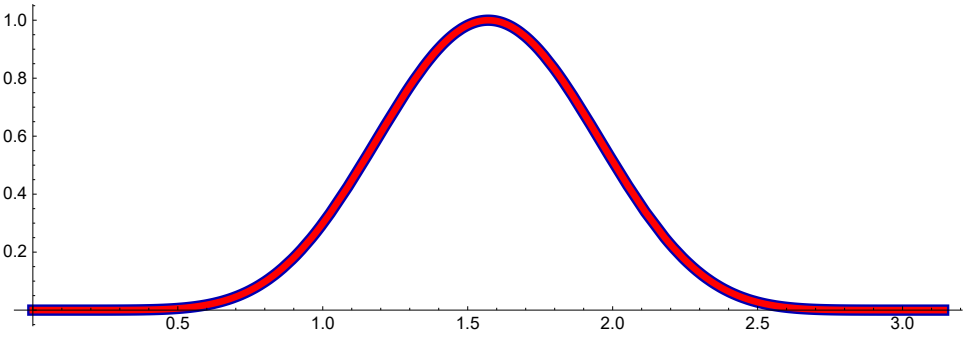
For $(\text{Sin}[x])^7$ we have

In[10]:=
```
Table[Integrate[Sin[x]^7 Sin[k x], {x, 0, Pi}]/(Pi/2), {k, 1, 20}]
```

Out[10]=
$$\left\{\frac{35}{64}, 0, -\frac{21}{64}, 0, \frac{7}{64}, 0, -\frac{1}{64}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\right\}$$

In[30]:=
```
Plot[{Sin[x]^7, 35/64 Sin[x] - 21/64 Sin[3 x] + 7/64 Sin[5 x] - 1/64 Sin[7 x]}, {x, 0, Pi}, PlotStyle →
  {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.007]}},
 AspectRatio → Automatic, ImageSize → 500]
```

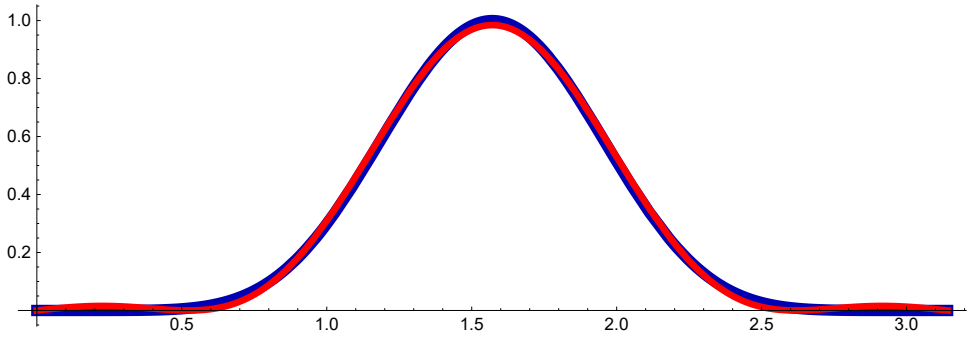
Out[30]=



Just to illustrate that even with three summands we get a good approximation for the blue function

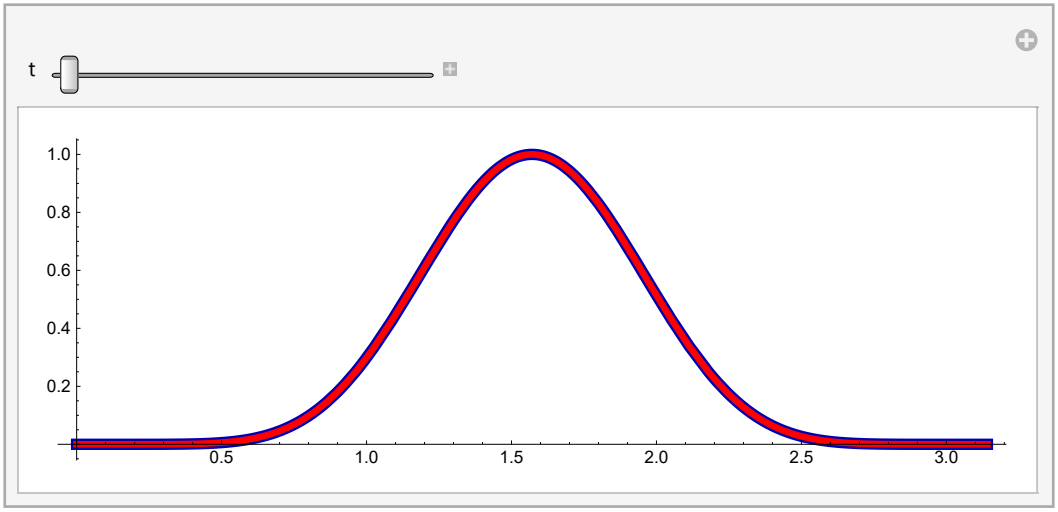
In[31]:=
```
Plot[{Sin[x]^7, 35/64 Sin[x] - 21/64 Sin[3 x] + 7/64 Sin[5 x]}, {x, 0, Pi}, PlotStyle →
  {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.007]}},
 AspectRatio → Automatic, ImageSize → 500]
```

Out[31]=



In[32]:=
```
Manipulate[Plot[{Sin[x]^7, 35/64 Exp[-t] Sin[x] - Exp[-9 t] 21/64 Sin[3 x] +
    7/64 Exp[-25 t] Sin[5 x] - 1/64 Exp[-49 t] Sin[7 x]}, {x, 0, Pi}, PlotStyle →
  {{RGBColor[0, 0, 0.7], Thickness[0.012]}, {RGBColor[1, 0, 0], Thickness[0.007]}},
 AspectRatio → Automatic, ImageSize → 500], {t, 0, 5}]
```

Out[32]=



Does the solution satisfy the given PDE?

In[16]:=
```
D[35/64 Exp[-t] Sin[x] - Exp[-9 t] 21/64 Sin[3 x] +
  7/64 Exp[-25 t] Sin[5 x] - 1/64 Exp[-49 t] Sin[7 x], t]
```

Out[16]=
$$-\frac{35}{64} e^{-t} \sin[x] + \frac{189}{64} e^{-9t} \sin[3x] - \frac{175}{64} e^{-25t} \sin[5x] + \frac{49}{64} e^{-49t} \sin[7x]$$

In[17]:=
```
D[35/64 Exp[-t] Sin[x] - Exp[-9 t] 21/64 Sin[3 x] +
  7/64 Exp[-25 t] Sin[5 x] - 1/64 Exp[-49 t] Sin[7 x], {x, 2}]
```

Out[17]=
$$-\frac{35}{64} e^{-t} \sin[x] + \frac{189}{64} e^{-9t} \sin[3x] - \frac{175}{64} e^{-25t} \sin[5x] + \frac{49}{64} e^{-49t} \sin[7x]$$

Are they equal?

In[18]:=
```
D[35/64 Exp[-t] Sin[x] - Exp[-9 t] 21/64 Sin[3 x] + 7/64 Exp[-25 t] Sin[5 x] - 1/64 Exp[-49 t] Sin[7 x],
  t] == D[35/64 Exp[-t] Sin[x] - Exp[-9 t] 21/64 Sin[3 x] +
  7/64 Exp[-25 t] Sin[5 x] - 1/64 Exp[-49 t] Sin[7 x], {x, 2}]
```

Out[18]=
```
True
```