ON THE VARIATION OF 3×3 STOCHASTIC MATRICES

BRANKO ĆURGUS AND ROBERT I. JEWETT

A *Markov* (or *stochastic*) *matrix* is a square matrix whose entries are non-negative and whose column sums are equal to 1. As in [1], since the term stochastic matrix seems to prevail in current literature we use it in the title. But, since a Markov matrix is a transition matrix of a Markov chain, we prefer the term Markov matrix and we use it from now on.

First we recall the definition of the variation from [1]. For a general $n\times n$ matrix A with columns $A_1,\ldots,A_n$ the *variation* (or *column variation*) of A is defined by:

$$\operatorname{var} A := \frac{1}{2} \max_{1\leq j,\,k\leq n} \bigl\| A_j - A_k \bigr\|.$$

Here $\|\cdot\|$ denotes the ℓ_1-norm of columns of A. That is, if X is a column with entries $x_1,\ldots,x_n$, then $\|X\| := |x_1| + \cdots + |x_1|$.

Next, we restate Propositions 3.2, 5.1 and 5.2 from [1] for Markov matrices.

Proposition 1. *If M is a Markov matrix, then the sequence* $\operatorname{var}(M^k), k\in\mathbb{N}$*, is non-increasing.*

Proposition 2. *If M is a Markov matrix, then* $\operatorname{var} M \leq 1$.

Proposition 3. *Let M be an $n\times n$ Markov matrix. Then the following two statements are equivalent.*

(i) *The strict inequality* $\operatorname{var} M < 1$ *holds.*
(ii) *For each $k, l \in \{1,\ldots,n\}$ there exists $j\in\{1,\ldots,m\}$ such that the j-th entries of the k-th and l-th columns of M are both positive.*

Remark 4. The easiest special case to verify (ii) in Proposition 3 is when a matrix has a row of positive entries.

All claims about the variation in the proof of the following theorem rely on Propositions 2 and 3. In fact the observation in Remark 4 is sufficient in most cases.

Theorem 5. *For a 3×3 Markov matrix M, the sequence $M^k, k\in\mathbb{N}$, converges to a projection of rank 1 if and only if* $\operatorname{var}(M^3) < 1$.

Proof. The entries of a Markov matrix are numbers form the interval $[0,1]$. In this proof we use $+$ for numbers in $(0,1)$. There are seven possible forms for a column of a 3×3 Markov matrix. Below we list them in the "alphabetic" order $(0 < + < 1)$ starting from the top entry:

$$\begin{bmatrix}0\\0\\1\end{bmatrix}, \quad \begin{bmatrix}0\\+\\+\end{bmatrix}, \quad \begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \begin{bmatrix}+\\0\\+\end{bmatrix}, \quad \begin{bmatrix}+\\+\\0\end{bmatrix}, \quad \begin{bmatrix}+\\+\\+\end{bmatrix}, \quad \begin{bmatrix}1\\0\\0\end{bmatrix}.$$

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Any of these columns can be one of the columns of a 3×3 Markov matrix. Therefore there are $7^3 = 343$ different forms for a 3×3 Markov matrix.

We used the computer algebra system *Mathematica* to verify the theorem for all 343 cases. In the appendix to this note we present 19 pages of *Mathematica* output which contains the list of all 343 different forms of 3×3 Markov matrices organized in the following way:

- Matrices are in the "alphabetic" order starting from the top-left entry and proceeding along columns.
- Each matrix is given a separate row of the list. Each row is numbered.

And now to the numbers that prove the theorem.

- If a power of a matrix has a variation less than 1, then we list the powers of the matrix until the corresponding variation is less than 1.

  There are 289 such matrices. A more detailed account is as follows:
  - There are 175 matrices with the variation less than 1. Except for the matrices in the rows 75, 81, 159, 177, 207, and 219, all such matrices have positive rows.
  - There are 108 matrices for which the square is the lowest power with the variation < 1. Interestingly, all squares here have positive rows.
  - There are 6 matrices for which the cube is the lowest power with the variation < 1. They are in the rows 24, 47, 56, 94, 103, and 126. Again, each of the six cubes has a positive row.
- If all the powers of a matrix have the variation equal to 1, then, by Theorem 4.2 and Remark 4.3 in [1], the spectrum of the matrix on the unit circle in the complex plane has the multiplicity greater than 1.

  There are 54 such matrices. In the list that follows we give the eigenvalues of such matrices. As before $+$ stands for a number in $(0, 1)$ and $-$ stands for a number in $(-1, 0)$. The symbol $+/-$ stands for any number in $(-1, 1)$.

  Here is a more detailed analysis of these matrices.
  - There are 6 matrices with all three eigenvalues on the unit circle. Those are 3×3 permutation matrices which are in the rows 21, 45, 105, 141, 297, and 309.
  - There are 27 matrices with 1 being an eigenvalue of geometric multiplicity 2 and the third eigenvalue in the interval $(-1, 1)$. For 9 matrices the third eigenvalue is 0; for 9 matrices the third eigenvalue is the interval $(0, 1)$; and for 6 matrices the third eigenvalue is the interval $(-1, 0)$. There are only 3 such matrices for which the sign of the third eigenvalue can not be determined from the form of the matrix. They are in the rows 165, 225, and 303.
  - There are 21 matrices whose spectrum on the unit circle consists of 1 and -1. There are 12 such matrices for which the third eigenvalue is 0. For the remaining 11 matrices the third eigenvalue is in the interval $(0, 1)$.

Since $289 + 54 = 343$, the theorem is proved. □

Remark 6. The method used in the proof of Theorem 5 can also be used for 4×4 Markov matrices. The following statement holds: For a 4×4 Markov matrix M, the sequence $M^k, k \in \mathbb{N}$, converges to a projection of rank 1 if and only if $\operatorname{var}(M^5) < 1$.

There are 15 possible forms for a column of a 4×4 Markov matrix. Hence, there are $15^4 = 50625$ different forms of 4×4 Markov matrices.

It took *Mathematica* 52.054 seconds to check all 15^4 matrices. The corresponding calculations for 3×3 Markov matrices took 0.241 seconds. The following table is a summary of the calculations for 4×4 Markov matrices:

a power has the variation < 1			the number of matrices
	the lowest power with the variation < 1	the number of matrices	
	1	17887	
	2	24948	
	3	3564	
	4	120	
	5	24	
total			46543
all powers have the variation $= 1$			4082
total			50625

With 18 matrices per page it would take 2812.5 pages to present these findings as we did for 3×3 Markov matrices.

Remark 7. Let n be a positive integer and consider $n\times n$ Markov matrices. There are $2^n - 1$ different forms for a column of an $n\times n$ Markov matrices. Therefore, there are $\bigl(2^n - 1\bigr)^n$ different forms of $n\times n$ Markov matrices.

Using the exponential interpolation on two timings cited in Remark 6 indicates that it would take approximately 3 hours of computer time to check all $n\times n$ Markov matrices for $n = 5$, 28 days for $n = 6$, and 506 years for $n = 7$.

Remark 8. Let n be a positive integer such that $n \geq 3$. In this remark we give a sparse $n\times n$ Markov matrix M_n for which the smallest p_n such that $\operatorname{var}\bigl(M_n^{p_n}\bigr) < 1$ is large. The definition of M_n below is suggested by the matrix in the 24th row of the list in the Appendix and the corresponding 4×4 Markov matrix.

Let $a \in (0,1)$. Let M_n be the $n\times n$ matrix with the entries a_{jk} where $j,k \in \{1,\ldots,n\}$ and

$$a_{jk} = \begin{cases} \left.\begin{array}{rl} 1 & \text{if } \ j+k=n+1 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \quad k < \lfloor n/2 \rfloor + 1 \\ \left.\begin{array}{rl} a & \text{if } \ j=1 \\ 0 & \text{if } \ 1<j<n \\ 1-a & \text{if } \ j=n \end{array}\right\} & \text{if } \quad k = \lfloor n/2 \rfloor + 1 \\ \left.\begin{array}{rl} 1 & \text{if } \ j+k=n+2 \\ 0 & \text{otherwise} \end{array}\right\} & \text{if } \quad k > \lfloor n/2 \rfloor + 1 \end{cases} .$$

For $n = 4, 5, 6$ we get the following matrices:

$$\begin{bmatrix} 0 & 0 & + & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & + & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & + & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & + & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & + & 0 & 0 \end{bmatrix}.$$

The characteristic polynomial of M_n is

$$\det(M_n - xI) = a(x-1) + x^n - x = (-1)^n (x-1)(a + x + \cdots + x^{n-1}).$$

Since all the roots of $a + x + \cdots + x^{n-1}$ are in the open unit disc, we conclude that a power of M_n must have the variation < 1. Again we used *Mathematica* to produce the following table of the lowest powers p_n for which $\operatorname{var}\big(M_n^{p_n}\big) < 1$:

n	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
p_n	3	5	9	13	19	25	33	41	51	61	73	85	99	113	129	145

Using [2] we found that the numbers $p_n, n < 19$, calculated by *Mathematica* are a part of the integer sequence, see [2, sequence A099392], given by the formula

$$p_n := \left\lfloor \frac{(n-1)^2}{2} \right\rfloor + 1, \quad n \in \mathbb{N}.$$

This leads to the following conjecture:

Conjecture 9. *Let n be a positive integer. For an $n \times n$ Markov matrix M, the sequence $M^k, k \in \mathbb{N}$, converges to a projection of rank 1 if and only if* $\operatorname{var}(M^{p_n}) < 1$.

The Appendix starts on the next page.

References

[1] B. Ćurgus and R. I. Jewett, Somewhat stochastic matrices, submitted for publication. Available at `http://myweb.facstaff.wwu.edu/curgus/papers.html`.

[2] N. J. A. Sloane, On-Line Encyclopedia of Integer Sequences, (August 2007) available at `http://www.research.att.com/~njas/sequences/`

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1. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & + \\ 1 & 1 & + \end{pmatrix}$

3. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

4. $\begin{pmatrix} 0 & 0 & + \\ 0 & 0 & 0 \\ 1 & 1 & + \end{pmatrix}$

5. $\begin{pmatrix} 0 & 0 & + \\ 0 & 0 & + \\ 1 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

6. $\begin{pmatrix} 0 & 0 & + \\ 0 & 0 & + \\ 1 & 1 & + \end{pmatrix}$

7. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

8. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & + & 0 \\ 1 & + & 1 \end{pmatrix}$

9. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & + & + \\ 1 & + & + \end{pmatrix}$

10. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & + & 1 \\ 1 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 1 & + & + \\ 0 & + & + \end{pmatrix}$

11. $\begin{pmatrix} 0 & 0 & + \\ 0 & + & 0 \\ 1 & + & + \end{pmatrix}$

12. $\begin{pmatrix} 0 & 0 & + \\ 0 & + & + \\ 1 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & + \\ 0 & + & + \end{pmatrix}$

13. $\begin{pmatrix} 0 & 0 & + \\ 0 & + & + \\ 1 & + & + \end{pmatrix}$

14. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & + & 0 \\ 1 & + & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

15. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, 0\}$

16. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

17. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

18. $\begin{pmatrix} 0 & 0 & + \\ 0 & 1 & 0 \\ 1 & 0 & + \end{pmatrix}$ the eigenvalues are $\{1, 1, -\}$

19. $\begin{pmatrix} 0 & 0 & + \\ 0 & 1 & + \\ 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & 1 & + \\ 0 & 0 & + \end{pmatrix}$

20. $\begin{pmatrix} 0 & 0 & + \\ 0 & 1 & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

21. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 1\}$

22. $\begin{pmatrix} 0 & + & 0 \\ 0 & 0 & 0 \\ 1 & + & 1 \end{pmatrix}$

23. $\begin{pmatrix} 0 & + & 0 \\ 0 & 0 & + \\ 1 & + & + \end{pmatrix}$

24. $\begin{pmatrix} 0 & + & 0 \\ 0 & 0 & 1 \\ 1 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & + \\ 1 & + & 0 \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ 0 & + & + \\ + & + & + \end{pmatrix}$

25. $\begin{pmatrix} 0 & + & + \\ 0 & 0 & 0 \\ 1 & + & + \end{pmatrix}$

26. $\begin{pmatrix} 0 & + & + \\ 0 & 0 & + \\ 1 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & 0 \\ 0 & + & + \end{pmatrix}$

27. $\begin{pmatrix} 0 & + & + \\ 0 & 0 & + \\ 1 & + & + \end{pmatrix}$

28. $\begin{pmatrix} 0 & + & 1 \\ 0 & 0 & 0 \\ 1 & + & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

29. $\begin{pmatrix} 0 & + & 0 \\ 0 & + & 0 \\ 1 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & + & 0 \\ 0 & + & 0 \\ 1 & + & 1 \end{pmatrix}$

30. $\begin{pmatrix} 0 & + & 0 \\ 0 & + & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} 0 & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

31. $\begin{pmatrix} 0 & + & 0 \\ 0 & + & 1 \\ 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & + & + \\ 1 & + & + \\ 0 & + & 0 \end{pmatrix}$

32. $\begin{pmatrix} 0 & + & + \\ 0 & + & 0 \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ 0 & + & 0 \\ + & + & + \end{pmatrix}$

33. $\begin{pmatrix} 0 & + & + \\ 0 & + & + \\ 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & + & + \end{pmatrix}$

34. $\begin{pmatrix} 0 & + & + \\ 0 & + & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

35. $\begin{pmatrix} 0 & + & 1 \\ 0 & + & 0 \\ 1 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

36. $\begin{pmatrix} 0 & + & 0 \\ 0 & + & 0 \\ 1 & + & 1 \end{pmatrix}$

37. $\begin{pmatrix} 0 & + & 0 \\ 0 & + & + \\ 1 & + & + \end{pmatrix}$

38. $\begin{pmatrix} 0 & + & 0 \\ 0 & + & 1 \\ 1 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & + & + \\ 1 & + & + \\ 0 & + & + \end{pmatrix}$

39. $\begin{pmatrix} 0 & + & + \\ 0 & + & 0 \\ 1 & + & + \end{pmatrix}$

40. $\begin{pmatrix} 0 & + & + \\ 0 & + & + \\ 1 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & + & + \end{pmatrix}$

41. $\begin{pmatrix} 0 & + & + \\ 0 & + & + \\ 1 & + & + \end{pmatrix}$

42. $\begin{pmatrix} 0 & + & 1 \\ 0 & + & 0 \\ 1 & + & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

43. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

44. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & + \\ + & 0 & + \\ + & 1 & + \end{pmatrix}$

45. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\left\{-\frac{1}{2} + \frac{i\sqrt{3}}{2}, -\frac{1}{2} - \frac{i\sqrt{3}}{2}, 1\right\}$

46. $\begin{pmatrix} 0 & 1 & + \\ 0 & 0 & 0 \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ 0 & 0 & 0 \\ + & 1 & + \end{pmatrix}$

47. $\begin{pmatrix} 0 & 1 & + \\ 0 & 0 & + \\ 1 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 0 & 0 \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & 0 & + \end{pmatrix}$

48. $\begin{pmatrix} 0 & 1 & + \\ 0 & 0 & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 0 & + \\ + & 1 & + \end{pmatrix}$

49. $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

50. $\begin{pmatrix} 0 & 0 & 0 \\ + & 0 & 0 \\ + & 1 & 1 \end{pmatrix}$

51. $\begin{pmatrix} 0 & 0 & 0 \\ + & 0 & + \\ + & 1 & + \end{pmatrix}$

52. $\begin{pmatrix} 0 & 0 & 0 \\ + & 0 & 1 \\ + & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

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54. $\begin{pmatrix} 0 & 0 & + \\ + & 0 & + \\ + & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & + \\ + & 0 & + \end{pmatrix}$

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59. $\begin{pmatrix} 0 & 0 & 0 \\ + & + & 1 \\ + & + & 0 \end{pmatrix}$

60. $\begin{pmatrix} 0 & 0 & + \\ + & + & 0 \\ + & + & + \end{pmatrix}$

61. $\begin{pmatrix} 0 & 0 & + \\ + & + & + \\ + & + & 0 \end{pmatrix}$

62. $\begin{pmatrix} 0 & 0 & + \\ + & + & + \\ + & + & + \end{pmatrix}$

63. $\begin{pmatrix} 0 & 0 & 1 \\ + & + & 0 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & + \\ + & + & + \end{pmatrix}$

64. $\begin{pmatrix} 0 & 0 & 0 \\ + & 1 & 0 \\ + & 0 & 1 \end{pmatrix}$ the eigenvalues are {1, 1, 0}

65. $\begin{pmatrix} 0 & 0 & 0 \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

66. $\begin{pmatrix} 0 & 0 & 0 \\ + & 1 & 1 \\ + & 0 & 0 \end{pmatrix}$

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68. $\begin{pmatrix} 0 & 0 & + \\ + & 1 & + \\ + & 0 & 0 \end{pmatrix}$

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77. $\begin{pmatrix} 0 & + & 1 \\ + & 0 & 0 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ 0 & + & + \\ + & + & + \end{pmatrix}$

78. $\begin{pmatrix} 0 & + & 0 \\ + & + & 0 \\ + & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

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80. $\begin{pmatrix} 0 & + & 0 \\ + & + & 1 \\ + & 0 & 0 \end{pmatrix}$

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84. $\begin{pmatrix} 0 & + & 1 \\ + & + & 0 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & + \\ 0 & + & + \end{pmatrix}$

85. $\begin{pmatrix} 0 & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

86. $\begin{pmatrix} 0 & + & 0 \\ + & + & + \\ + & + & + \end{pmatrix}$

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93. $\begin{pmatrix} 0 & 1 & 0 \\ + & 0 & + \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & + & + \\ + & + & + \end{pmatrix}$

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95. $\begin{pmatrix} 0 & 1 & + \\ + & 0 & 0 \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ 0 & + & + \\ + & + & + \end{pmatrix}$

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98. $\begin{pmatrix} 0 & 1 & 1 \\ + & 0 & 0 \\ + & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

99. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

100. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & + \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 0 & + & + \\ 1 & + & + \end{pmatrix}$

101. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

102. $\begin{pmatrix} 0 & 0 & + \\ 1 & 0 & 0 \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} 0 & + & + \\ 0 & 0 & + \\ 1 & + & + \end{pmatrix}$

103. $\begin{pmatrix} 0 & 0 & + \\ 1 & 0 & + \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & + & 0 \\ 0 & + & + \\ 1 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & + & + \\ 0 & + & + \end{pmatrix}$

104. $\begin{pmatrix} 0 & 0 & + \\ 1 & 0 & + \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} 0 & + & + \\ 0 & + & + \\ 1 & + & + \end{pmatrix}$

105. $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\left\{-\frac{1}{2} + \frac{i\sqrt{3}}{2}, -\frac{1}{2} - \frac{i\sqrt{3}}{2}, 1\right\}$

106. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & + & 0 \\ 0 & + & 1 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

107. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & + & + \\ 0 & + & + \end{pmatrix}$

108. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & + & 1 \\ 0 & + & 0 \end{pmatrix}$

109. $\begin{pmatrix} 0 & 0 & + \\ 1 & + & 0 \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} 0 & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

110. $\begin{pmatrix} 0 & 0 & + \\ 1 & + & + \\ 0 & + & 0 \end{pmatrix}$

111. $\begin{pmatrix} 0 & 0 & + \\ 1 & + & + \\ 0 & + & + \end{pmatrix}$

112. $\begin{pmatrix} 0 & 0 & 1 \\ 1 & + & 0 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & + & 0 \\ + & + & 1 \\ + & + & 0 \end{pmatrix}$

113. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are {1, 1, 0}

114. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & + \\ 0 & 0 & + \end{pmatrix}$

115. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

116. $\begin{pmatrix} 0 & 0 & + \\ 1 & 1 & 0 \\ 0 & 0 & + \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & + \\ 1 & 1 & + \\ 0 & 0 & + \end{pmatrix}$

117. $\begin{pmatrix} 0 & 0 & + \\ 1 & 1 & + \\ 0 & 0 & 0 \end{pmatrix}$

118. $\begin{pmatrix} 0 & 0 & + \\ 1 & 1 & + \\ 0 & 0 & + \end{pmatrix}$

119. $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

120. $\begin{pmatrix} 0 & + & 0 \\ 1 & 0 & 0 \\ 0 & + & 1 \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ 0 & + & 0 \\ + & + & 1 \end{pmatrix}$

121. $\begin{pmatrix} 0 & + & 0 \\ 1 & 0 & + \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ 0 & + & + \\ + & + & + \end{pmatrix}$

122. $\begin{pmatrix} 0 & + & 0 \\ 1 & 0 & 1 \\ 0 & + & 0 \end{pmatrix}$ the eigenvalues are {-1, 1, 0}

123. $\begin{pmatrix} 0 & + & + \\ 1 & 0 & 0 \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & + & + \end{pmatrix}$

124. $\begin{pmatrix} 0 & + & + \\ 1 & 0 & + \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & 0 & + \end{pmatrix}$

125. $\begin{pmatrix} 0 & + & + \\ 1 & 0 & + \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & + & + \end{pmatrix}$

126. $\begin{pmatrix} 0 & + & 1 \\ 1 & 0 & 0 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ 0 & + & 1 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & 0 \\ 0 & + & + \end{pmatrix}$

127. $\begin{pmatrix} 0 & + & 0 \\ 1 & + & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, -\}$

128. $\begin{pmatrix} 0 & + & 0 \\ 1 & + & + \\ 0 & 0 & + \end{pmatrix}$

129. $\begin{pmatrix} 0 & + & 0 \\ 1 & + & 1 \\ 0 & 0 & 0 \end{pmatrix}$

130. $\begin{pmatrix} 0 & + & + \\ 1 & + & 0 \\ 0 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & 0 & + \end{pmatrix}$

131. $\begin{pmatrix} 0 & + & + \\ 1 & + & + \\ 0 & 0 & 0 \end{pmatrix}$

132. $\begin{pmatrix} 0 & + & + \\ 1 & + & + \\ 0 & 0 & + \end{pmatrix}$

133. $\begin{pmatrix} 0 & + & 1 \\ 1 & + & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 1 \\ 0 & 0 & 0 \end{pmatrix}$

134. $\begin{pmatrix} 0 & + & 0 \\ 1 & + & 0 \\ 0 & + & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

135. $\begin{pmatrix} 0 & + & 0 \\ 1 & + & + \\ 0 & + & + \end{pmatrix}$

136. $\begin{pmatrix} 0 & + & 0 \\ 1 & + & 1 \\ 0 & + & 0 \end{pmatrix}$

137. $\begin{pmatrix} 0 & + & + \\ 1 & + & 0 \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

138. $\begin{pmatrix} 0 & + & + \\ 1 & + & + \\ 0 & + & 0 \end{pmatrix}$

139. $\begin{pmatrix} 0 & + & + \\ 1 & + & + \\ 0 & + & + \end{pmatrix}$

140. $\begin{pmatrix} 0 & + & 1 \\ 1 & + & 0 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 1 \\ + & + & 0 \end{pmatrix}$

141. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 1\}$

142. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & + \\ 0 & 0 & + \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

143. $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

144. $\begin{pmatrix} 0 & 1 & + \\ 1 & 0 & 0 \\ 0 & 0 & + \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

145. $\begin{pmatrix} 0 & 1 & + \\ 1 & 0 & + \\ 0 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

146. $\begin{pmatrix} 0 & 1 & + \\ 1 & 0 & + \\ 0 & 0 & + \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

147. $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 0\}$

148. $\begin{pmatrix} + & 0 & 0 \\ 0 & 0 & 0 \\ + & 1 & 1 \end{pmatrix}$

149. $\begin{pmatrix} + & 0 & 0 \\ 0 & 0 & + \\ + & 1 & + \end{pmatrix}$

150. $\begin{pmatrix} + & 0 & 0 \\ 0 & 0 & 1 \\ + & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

151. $\begin{pmatrix} + & 0 & + \\ 0 & 0 & 0 \\ + & 1 & + \end{pmatrix}$

152. $\begin{pmatrix} + & 0 & + \\ 0 & 0 & + \\ + & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & 0 \\ + & 0 & + \end{pmatrix}$

153. $\begin{pmatrix} + & 0 & + \\ 0 & 0 & + \\ + & 1 & + \end{pmatrix}$

154. $\begin{pmatrix} + & 0 & 1 \\ 0 & 0 & 0 \\ + & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 1 & + \\ 0 & 0 & 0 \\ + & 0 & + \end{pmatrix}$

155. $\begin{pmatrix} + & 0 & 0 \\ 0 & + & 0 \\ + & + & 1 \end{pmatrix}$

156. $\begin{pmatrix} + & 0 & 0 \\ 0 & + & + \\ + & + & + \end{pmatrix}$

157. $\begin{pmatrix} + & 0 & 0 \\ 0 & + & 1 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & + & + \\ + & + & + \end{pmatrix}$

158. $\begin{pmatrix} + & 0 & + \\ 0 & + & 0 \\ + & + & + \end{pmatrix}$

159. $\begin{pmatrix} + & 0 & + \\ 0 & + & + \\ + & + & 0 \end{pmatrix}$

160. $\begin{pmatrix} + & 0 & + \\ 0 & + & + \\ + & + & + \end{pmatrix}$

161. $\begin{pmatrix} + & 0 & 1 \\ 0 & + & 0 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ 0 & + & 0 \\ + & + & + \end{pmatrix}$

162. $\begin{pmatrix} + & 0 & 0 \\ 0 & 1 & 0 \\ + & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, +\}$

163. $\begin{pmatrix} + & 0 & 0 \\ 0 & 1 & + \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

164. $\begin{pmatrix} + & 0 & 0 \\ 0 & 1 & 1 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & 1 & 1 \\ + & 0 & 0 \end{pmatrix}$

165. $\begin{pmatrix} + & 0 & + \\ 0 & 1 & 0 \\ + & 0 & + \end{pmatrix}$ the eigenvalues are {1, 1, +/−}

166. $\begin{pmatrix} + & 0 & + \\ 0 & 1 & + \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

167. $\begin{pmatrix} + & 0 & + \\ 0 & 1 & + \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

168. $\begin{pmatrix} + & 0 & 1 \\ 0 & 1 & 0 \\ + & 0 & 0 \end{pmatrix}$ the eigenvalues are {1, 1, −}

169. $\begin{pmatrix} + & + & 0 \\ 0 & 0 & 0 \\ + & + & 1 \end{pmatrix}$

170. $\begin{pmatrix} + & + & 0 \\ 0 & 0 & + \\ + & + & + \end{pmatrix}$

171. $\begin{pmatrix} + & + & 0 \\ 0 & 0 & 1 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & 0 \\ + & + & + \end{pmatrix}$

172. $\begin{pmatrix} + & + & + \\ 0 & 0 & 0 \\ + & + & + \end{pmatrix}$

173. $\begin{pmatrix} + & + & + \\ 0 & 0 & + \\ + & + & 0 \end{pmatrix}$

174. $\begin{pmatrix} + & + & + \\ 0 & 0 & + \\ + & + & + \end{pmatrix}$

175. $\begin{pmatrix} + & + & 1 \\ 0 & 0 & 0 \\ + & + & 0 \end{pmatrix}$

176. $\begin{pmatrix} + & + & 0 \\ 0 & + & 0 \\ + & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ 0 & + & 0 \\ + & + & 1 \end{pmatrix}$

177. $\begin{pmatrix} + & + & 0 \\ 0 & + & + \\ + & 0 & + \end{pmatrix}$

178. $\begin{pmatrix} + & + & 0 \\ 0 & + & 1 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & 0 \end{pmatrix}$

179. $\begin{pmatrix} + & + & + \\ 0 & + & 0 \\ + & 0 & + \end{pmatrix}$

180. $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & 0 & 0 \end{pmatrix}$

181. $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & 0 & + \end{pmatrix}$

182. $\begin{pmatrix} + & + & 1 \\ 0 & + & 0 \\ + & 0 & 0 \end{pmatrix}$

183. $\begin{pmatrix} + & + & 0 \\ 0 & + & 0 \\ + & + & 1 \end{pmatrix}$

184. $\begin{pmatrix} + & + & 0 \\ 0 & + & + \\ + & + & + \end{pmatrix}$

185. $\begin{pmatrix} + & + & 0 \\ 0 & + & 1 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

186. $\begin{pmatrix} + & + & + \\ 0 & + & 0 \\ + & + & + \end{pmatrix}$

187. $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & + & 0 \end{pmatrix}$

188. $\begin{pmatrix} + & + & + \\ 0 & + & + \\ + & + & + \end{pmatrix}$

189. $\begin{pmatrix} + & + & 1 \\ 0 & + & 0 \\ + & + & 0 \end{pmatrix}$

190. $\begin{pmatrix} + & 1 & 0 \\ 0 & 0 & 0 \\ + & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ 0 & 0 & 0 \\ + & + & 1 \end{pmatrix}$

191. $\begin{pmatrix} + & 1 & 0 \\ 0 & 0 & + \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & 0 & + \\ + & + & + \end{pmatrix}$

192. $\begin{pmatrix} + & 1 & 0 \\ 0 & 0 & 1 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 1 \\ + & 0 & 0 \\ + & + & 0 \end{pmatrix}$

193. $\begin{pmatrix} + & 1 & + \\ 0 & 0 & 0 \\ + & 0 & + \end{pmatrix}$

194. $\begin{pmatrix} + & 1 & + \\ 0 & 0 & + \\ + & 0 & 0 \end{pmatrix}$

195. $\begin{pmatrix} + & 1 & + \\ 0 & 0 & + \\ + & 0 & + \end{pmatrix}$

196. $\begin{pmatrix} + & 1 & 1 \\ 0 & 0 & 0 \\ + & 0 & 0 \end{pmatrix}$

197. $\begin{pmatrix} + & 0 & 0 \\ + & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & 0 & 0 \\ + & 1 & 1 \end{pmatrix}$

198. $\begin{pmatrix} + & 0 & 0 \\ + & 0 & + \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & + & + \\ + & + & + \end{pmatrix}$

199. $\begin{pmatrix} + & 0 & 0 \\ + & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

200. $\begin{pmatrix} + & 0 & + \\ + & 0 & 0 \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & 0 & + \\ + & + & + \end{pmatrix}$

201. $\begin{pmatrix} + & 0 & + \\ + & 0 & + \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & 0 & + \end{pmatrix}$

202. $\begin{pmatrix} + & 0 & + \\ + & 0 & + \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

203. $\begin{pmatrix} + & 0 & 1 \\ + & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 1 & + \\ + & 0 & + \\ + & 0 & 0 \end{pmatrix}$

204. $\begin{pmatrix} + & 0 & 0 \\ + & + & 0 \\ 0 & + & 1 \end{pmatrix}$ $\begin{pmatrix} + & 0 & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

205. $\begin{pmatrix} + & 0 & 0 \\ + & + & + \\ 0 & + & + \end{pmatrix}$

206. $\begin{pmatrix} + & 0 & 0 \\ + & + & 1 \\ 0 & + & 0 \end{pmatrix}$

207. $\begin{pmatrix} + & 0 & + \\ + & + & 0 \\ 0 & + & + \end{pmatrix}$

208. $\begin{pmatrix} + & 0 & + \\ + & + & + \\ 0 & + & 0 \end{pmatrix}$

209. $\begin{pmatrix} + & 0 & + \\ + & + & + \\ 0 & + & + \end{pmatrix}$

210. $\begin{pmatrix} + & 0 & 1 \\ + & + & 0 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & 0 \end{pmatrix}$

211. $\begin{pmatrix} + & 0 & 0 \\ + & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, +\}$

212. $\begin{pmatrix} + & 0 & 0 \\ + & 1 & + \\ 0 & 0 & + \end{pmatrix}$

213. $\begin{pmatrix} + & 0 & 0 \\ + & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

214. $\begin{pmatrix} + & 0 & + \\ + & 1 & 0 \\ 0 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ 0 & 0 & + \end{pmatrix}$

215. $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ 0 & 0 & 0 \end{pmatrix}$

216. $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ 0 & 0 & + \end{pmatrix}$

217. $\begin{pmatrix} + & 0 & 1 \\ + & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ 0 & 0 & 0 \end{pmatrix}$

218. $\begin{pmatrix} + & + & 0 \\ + & 0 & 0 \\ 0 & + & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

219. $\begin{pmatrix} + & + & 0 \\ + & 0 & + \\ 0 & + & + \end{pmatrix}$

220. $\begin{pmatrix} + & + & 0 \\ + & 0 & 1 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & 0 \\ + & 0 & + \end{pmatrix}$

221. $\begin{pmatrix} + & + & + \\ + & 0 & 0 \\ 0 & + & + \end{pmatrix}$

222. $\begin{pmatrix} + & + & + \\ + & 0 & + \\ 0 & + & 0 \end{pmatrix}$

223. $\begin{pmatrix} + & + & + \\ + & 0 & + \\ 0 & + & + \end{pmatrix}$

224. $\begin{pmatrix} + & + & 1 \\ + & 0 & 0 \\ 0 & + & 0 \end{pmatrix}$

225. $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are {1, 1, +/−}

226. $\begin{pmatrix} + & + & 0 \\ + & + & + \\ 0 & 0 & + \end{pmatrix}$

227. $\begin{pmatrix} + & + & 0 \\ + & + & 1 \\ 0 & 0 & 0 \end{pmatrix}$

228. $\begin{pmatrix} + & + & + \\ + & + & 0 \\ 0 & 0 & + \end{pmatrix}$

229. $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & 0 & 0 \end{pmatrix}$

230. $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & 0 & + \end{pmatrix}$

231. $\begin{pmatrix} + & + & 1 \\ + & + & 0 \\ 0 & 0 & 0 \end{pmatrix}$

232. $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ 0 & + & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

233. $\begin{pmatrix} + & + & 0 \\ + & + & + \\ 0 & + & + \end{pmatrix}$

234. $\begin{pmatrix} + & + & 0 \\ + & + & 1 \\ 0 & + & 0 \end{pmatrix}$

235. $\begin{pmatrix} + & + & + \\ + & + & 0 \\ 0 & + & + \end{pmatrix}$

236. $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & + & 0 \end{pmatrix}$

237. $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & + & + \end{pmatrix}$

238. $\begin{pmatrix} + & + & 1 \\ + & + & 0 \\ 0 & + & 0 \end{pmatrix}$

239. $\begin{pmatrix} + & 1 & 0 \\ + & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, -\}$

240. $\begin{pmatrix} + & 1 & 0 \\ + & 0 & + \\ 0 & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ 0 & 0 & + \end{pmatrix}$

241. $\begin{pmatrix} + & 1 & 0 \\ + & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 1 \\ + & + & 0 \\ 0 & 0 & 0 \end{pmatrix}$

242. $\begin{pmatrix} + & 1 & + \\ + & 0 & 0 \\ 0 & 0 & + \end{pmatrix}$

243. $\begin{pmatrix} + & 1 & + \\ + & 0 & + \\ 0 & 0 & 0 \end{pmatrix}$

244. $\begin{pmatrix} + & 1 & + \\ + & 0 & + \\ 0 & 0 & + \end{pmatrix}$

245. $\begin{pmatrix} + & 1 & 1 \\ + & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

246. $\begin{pmatrix} + & 0 & 0 \\ + & 0 & 0 \\ + & 1 & 1 \end{pmatrix}$

247. $\begin{pmatrix} + & 0 & 0 \\ + & 0 & + \\ + & 1 & + \end{pmatrix}$

248. $\begin{pmatrix} + & 0 & 0 \\ + & 0 & 1 \\ + & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, +\}$

249. $\begin{pmatrix} + & 0 & + \\ + & 0 & 0 \\ + & 1 & + \end{pmatrix}$

250. $\begin{pmatrix} + & 0 & + \\ + & 0 & + \\ + & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & 0 & + \end{pmatrix}$

251. $\begin{pmatrix} + & 0 & + \\ + & 0 & + \\ + & 1 & + \end{pmatrix}$

252. $\begin{pmatrix} + & 0 & 1 \\ + & 0 & 0 \\ + & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 1 & + \\ + & 0 & + \\ + & 0 & + \end{pmatrix}$

253. $\begin{pmatrix} + & 0 & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

254. $\begin{pmatrix} + & 0 & 0 \\ + & + & + \\ + & + & + \end{pmatrix}$

255. $\begin{pmatrix} + & 0 & 0 \\ + & + & 1 \\ + & + & 0 \end{pmatrix}$

256. $\begin{pmatrix} + & 0 & + \\ + & + & 0 \\ + & + & + \end{pmatrix}$

257. $\begin{pmatrix} + & 0 & + \\ + & + & + \\ + & + & 0 \end{pmatrix}$

258. $\begin{pmatrix} + & 0 & + \\ + & + & + \\ + & + & + \end{pmatrix}$

259. $\begin{pmatrix} + & 0 & 1 \\ + & + & 0 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

260. $\begin{pmatrix} + & 0 & 0 \\ + & 1 & 0 \\ + & 0 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, +\}$

261. $\begin{pmatrix} + & 0 & 0 \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

262. $\begin{pmatrix} + & 0 & 0 \\ + & 1 & 1 \\ + & 0 & 0 \end{pmatrix}$

263. $\begin{pmatrix} + & 0 & + \\ + & 1 & 0 \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

264. $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & 0 \end{pmatrix}$

265. $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

266. $\begin{pmatrix} + & 0 & 1 \\ + & 1 & 0 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & 0 & + \\ + & 1 & + \\ + & 0 & + \end{pmatrix}$

267. $\begin{pmatrix} + & + & 0 \\ + & 0 & 0 \\ + & + & 1 \end{pmatrix}$

268. $\begin{pmatrix} + & + & 0 \\ + & 0 & + \\ + & + & + \end{pmatrix}$

269. $\begin{pmatrix} + & + & 0 \\ + & 0 & 1 \\ + & + & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & 0 \\ + & + & + \end{pmatrix}$

270. $\begin{pmatrix} + & + & + \\ + & 0 & 0 \\ + & + & + \end{pmatrix}$

271. $\begin{pmatrix} + & + & + \\ + & 0 & + \\ + & + & 0 \end{pmatrix}$

272. $\begin{pmatrix} + & + & + \\ + & 0 & + \\ + & + & + \end{pmatrix}$

273. $\begin{pmatrix} + & + & 1 \\ + & 0 & 0 \\ + & + & 0 \end{pmatrix}$

274. $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

275. $\begin{pmatrix} + & + & 0 \\ + & + & + \\ + & 0 & + \end{pmatrix}$

276. $\begin{pmatrix} + & + & 0 \\ + & + & 1 \\ + & 0 & 0 \end{pmatrix}$

277. $\begin{pmatrix} + & + & + \\ + & + & 0 \\ + & 0 & + \end{pmatrix}$

278. $\begin{pmatrix} + & + & + \\ + & + & + \\ + & 0 & 0 \end{pmatrix}$

279. $\begin{pmatrix} + & + & + \\ + & + & + \\ + & 0 & + \end{pmatrix}$

280. $\begin{pmatrix} + & + & 1 \\ + & + & 0 \\ + & 0 & 0 \end{pmatrix}$

281. $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

282. $\begin{pmatrix} + & + & 0 \\ + & + & + \\ + & + & + \end{pmatrix}$

283. $\begin{pmatrix} + & + & 0 \\ + & + & 1 \\ + & + & 0 \end{pmatrix}$

284. $\begin{pmatrix} + & + & + \\ + & + & 0 \\ + & + & + \end{pmatrix}$

285. $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & 0 \end{pmatrix}$

286. $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

287. $\begin{pmatrix} + & + & 1 \\ + & + & 0 \\ + & + & 0 \end{pmatrix}$

288. $\begin{pmatrix} + & 1 & 0 \\ + & 0 & 0 \\ + & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} + & + & 0 \\ + & + & 0 \\ + & + & 1 \end{pmatrix}$

289. $\begin{pmatrix} + & 1 & 0 \\ + & 0 & + \\ + & 0 & + \end{pmatrix}$ $\begin{pmatrix} + & + & + \\ + & + & + \\ + & + & + \end{pmatrix}$

290. $\begin{pmatrix} + & 1 & 0 \\ + & 0 & 1 \\ + & 0 & 0 \end{pmatrix}$ $\begin{pmatrix} + & + & 1 \\ + & + & 0 \\ + & + & 0 \end{pmatrix}$

291. $\begin{pmatrix} + & 1 & + \\ + & 0 & 0 \\ + & 0 & + \end{pmatrix}$

292. $\begin{pmatrix} + & 1 & + \\ + & 0 & + \\ + & 0 & 0 \end{pmatrix}$

293. $\begin{pmatrix} + & 1 & + \\ + & 0 & + \\ + & 0 & + \end{pmatrix}$

294. $\begin{pmatrix} + & 1 & 1 \\ + & 0 & 0 \\ + & 0 & 0 \end{pmatrix}$

295. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, 0\}$

296. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & + \\ 0 & 1 & + \end{pmatrix}$ the eigenvalues are $\{1, 1, -\}$

297. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ the eigenvalues are $\{-1, 1, 1\}$

298. $\begin{pmatrix} 1 & 0 & + \\ 0 & 0 & 0 \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & 0 & 0 \\ 0 & + & + \end{pmatrix}$

299. $\begin{pmatrix} 1 & 0 & + \\ 0 & 0 & + \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & 0 \\ 0 & 0 & + \end{pmatrix}$

300. $\begin{pmatrix} 1 & 0 & + \\ 0 & 0 & + \\ 0 & 1 & + \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & + \\ 0 & + & + \end{pmatrix}$

301. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

302. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & + & 0 \\ 0 & + & 1 \end{pmatrix}$ the eigenvalues are $\{1, 1, +\}$

303. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & + & + \\ 0 & + & + \end{pmatrix}$ the eigenvalues are $\{1, 1, +/-\}$

304. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & + & 1 \\ 0 & + & 0 \end{pmatrix}$ the eigenvalues are $\{1, 1, -\}$

305. $\begin{pmatrix} 1 & 0 & + \\ 0 & + & 0 \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & 0 \\ 0 & + & + \end{pmatrix}$

306. $\begin{pmatrix} 1 & 0 & + \\ 0 & + & + \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & + \\ 0 & + & + \end{pmatrix}$

307. $\begin{pmatrix} 1 & 0 & + \\ 0 & + & + \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & + \\ 0 & + & + \end{pmatrix}$

308. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & + & 0 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & + & 1 \\ 0 & + & 0 \\ 0 & + & 0 \end{pmatrix}$

309. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are {1, 1, 1}

310. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & + \\ 0 & 0 & + \end{pmatrix}$ the eigenvalues are {1, 1, +}

311. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ the eigenvalues are {1, 1, 0}

312. $\begin{pmatrix} 1 & 0 & + \\ 0 & 1 & 0 \\ 0 & 0 & + \end{pmatrix}$ the eigenvalues are {1, 1, +}

313. $\begin{pmatrix} 1 & 0 & + \\ 0 & 1 & + \\ 0 & 0 & 0 \end{pmatrix}$ the eigenvalues are {1, 1, 0}

314. $\begin{pmatrix} 1 & 0 & + \\ 0 & 1 & + \\ 0 & 0 & + \end{pmatrix}$ the eigenvalues are {1, 1, +}

315. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ the eigenvalues are {1, 1, 0}

316. $\begin{pmatrix} 1 & + & 0 \\ 0 & 0 & 0 \\ 0 & + & 1 \end{pmatrix}$ the eigenvalues are {1, 1, 0}

317. $\begin{pmatrix} 1 & + & 0 \\ 0 & 0 & + \\ 0 & + & + \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & + \\ 0 & + & + \end{pmatrix}$

318. $\begin{pmatrix} 1 & + & 0 \\ 0 & 0 & 1 \\ 0 & + & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & 0 \\ 0 & 0 & + \end{pmatrix}$

319. $\begin{pmatrix} 1 & + & + \\ 0 & 0 & 0 \\ 0 & + & + \end{pmatrix}$

320. $\begin{pmatrix} 1 & + & + \\ 0 & 0 & + \\ 0 & + & 0 \end{pmatrix}$

321. $\begin{pmatrix} 1 & + & + \\ 0 & 0 & + \\ 0 & + & + \end{pmatrix}$

322. $\begin{pmatrix} 1 & + & 1 \\ 0 & 0 & 0 \\ 0 & + & 0 \end{pmatrix}$

323. $\begin{pmatrix} 1 & + & 0 \\ 0 & + & 0 \\ 0 & 0 & 1 \end{pmatrix}$ the eigenvalues are {1, 1, +}

324. $\begin{pmatrix} 1 & + & 0 \\ 0 & + & + \\ 0 & 0 & + \end{pmatrix}$ $\begin{pmatrix} 1 & + & + \\ 0 & + & + \\ 0 & 0 & + \end{pmatrix}$

325. $\begin{pmatrix}1 & + & 0\\ 0 & + & 1\\ 0 & 0 & 0\end{pmatrix}$ $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & 0 & 0\end{pmatrix}$

326. $\begin{pmatrix}1 & + & +\\ 0 & + & 0\\ 0 & 0 & +\end{pmatrix}$

327. $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & 0 & 0\end{pmatrix}$

328. $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & 0 & +\end{pmatrix}$

329. $\begin{pmatrix}1 & + & 1\\ 0 & + & 0\\ 0 & 0 & 0\end{pmatrix}$

330. $\begin{pmatrix}1 & + & 0\\ 0 & + & 0\\ 0 & + & 1\end{pmatrix}$ the eigenvalues are $\{1, 1, +\}$

331. $\begin{pmatrix}1 & + & 0\\ 0 & + & +\\ 0 & + & +\end{pmatrix}$ $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & + & +\end{pmatrix}$

332. $\begin{pmatrix}1 & + & 0\\ 0 & + & 1\\ 0 & + & 0\end{pmatrix}$ $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & + & +\end{pmatrix}$

333. $\begin{pmatrix}1 & + & +\\ 0 & + & 0\\ 0 & + & +\end{pmatrix}$

334. $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & + & 0\end{pmatrix}$

335. $\begin{pmatrix}1 & + & +\\ 0 & + & +\\ 0 & + & +\end{pmatrix}$

336. $\begin{pmatrix}1 & + & 1\\ 0 & + & 0\\ 0 & + & 0\end{pmatrix}$

337. $\begin{pmatrix}1 & 1 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix}$ the eigenvalues are $\{1, 1, 0\}$

338. $\begin{pmatrix}1 & 1 & 0\\ 0 & 0 & +\\ 0 & 0 & +\end{pmatrix}$ $\begin{pmatrix}1 & 1 & +\\ 0 & 0 & +\\ 0 & 0 & +\end{pmatrix}$

339. $\begin{pmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$ $\begin{pmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$

340. $\begin{pmatrix}1 & 1 & +\\ 0 & 0 & 0\\ 0 & 0 & +\end{pmatrix}$

341. $\begin{pmatrix}1 & 1 & +\\ 0 & 0 & +\\ 0 & 0 & 0\end{pmatrix}$

342. $\begin{pmatrix}1 & 1 & +\\ 0 & 0 & +\\ 0 & 0 & +\end{pmatrix}$

343. $\begin{pmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}$