THE MATHEMATICS DEPARTMENT PRESENTS

A MATHEMATICS COLLOQUIUM

FRIDAY, October 9, 2015
BOND HALL 225
4:00 pm

Title: Investigation of Crouzeix’s Conjecture via Nonsmooth Optimization

Speaker: Michael Overton, Courant Institute of Mathematical Sciences, NYU

Abstract: M. Crouzeix’s 2004 conjecture concerns the relationship between $\|p\|_{W(A)}$, the norm of a polynomial $p$ on $W(A)$, the field of values of a matrix $A$, and $\|p(A)\|_2$, the operator norm of the matrix $p(A)$. We use nonsmooth optimization to investigate the conjecture numerically, using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method to search for local minimizers of the “Crouzeix ratio” $\|p\|_{W(A)}/\|p(A)\|_2$ and Chebfun to compute the boundary of the field of values. The conjecture states that the globally minimal value of the Crouzeix ratio is 1/2. We present numerical results that lead to some theorems and further conjectures about globally and locally minimal values of the Crouzeix ratio when varying only $A$ (of given order, with $p$ fixed) or varying only $p$ (of given degree, with $A$ fixed), as well as locally minimal values of the ratio when minimizing over all $p$ and $A$. All the computations strongly support the truth of Crouzeix’s conjecture. This is joint work with Anne Greenbaum and Adrian Lewis.

Refreshments will precede the talk at 3:30pm in Bond Hall 300